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Actuator fault detection and estimation for the Lur'e differential inclusion system ☆

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ABSTRACT

This paper deals with actuator fault detection and estimation for the Lur'e differential inclusion system. An adaptive full-order observer is used to detect the occurrence of the actuator fault. Then, based on a reduced-order observer, an approach to estimate the actuator fault is presented. A simulation of rotor system is given to illustrate the effectiveness of the proposed method.

1. Introduction

Recently, the investigation of the Lur'e differential inclusion (DI) system has become a hot topic in the field of control [1–8]. The research motivation originates from the need to analyze the systems with non-smooth or discontinuous behavior, such as control dynamic systems with Coulomb friction, circuits systems with ideal diode, neural networks with discontinuous neuron activations and so on. In the Lur'e DI system, the set-valued mapping is only set-valued on a countable set of points and continuous on the other points. The current research mainly focuses on two aspects: one is the stabilization problem [3]. Under the extension of a Popov-like criterion, [3] designed a state feedback law to stabilize the Lur'e DI system. The other is the observer design [4–8]. [4,5] presented the observer design method for the Lur'e DI system by passive approach, it should be noted that [5] verified the well-posed property of the observer. [6] proved the existence of the reduce-order observer under the same conditions as that in [5]. By using the theory of adaptive observer, [7] designed an adaptive observer for the Lur'e DI system with uncertain parameters. [8] considered the non-fragile observer with disturbance attenuation for the Lur'e DI system. Besides the mentioned references, there are also other works on observers for the Lur'e DI system, such as [9,10].

In many real systems, the problem of fault detection and isolation (FDI) is very important because the actuator or sensor fault always arises. Many approaches have been well developed for FDI, such as neural-network-based method [11,12], system identification method [13,14], parity relations approach [15,16], the observer-based method [17–27]. Among these approaches, observer-based method has been studied extensively and proved to be one of the most effective method. By the

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theory of eigenstructure assignment, full-order observer was constructed for FDI [17]. Sliding mode observer has been used successfully in FDI context by Edwards et al. [18], then it was extended to study the system with sensor fault and uncertainty respectively by Tan and Edwards [19,20]. By using the theory of differential geometry, nonlinear observer has been designed for FDI of nonlinear systems [21]. For a class of nonlinear systems with uncertain parameters, the adaptive observer was employed for FDI [22]. Fault estimation is also important in FDI problem, because the size and characteristics of the fault can be determined if the fault is estimated. Based on the detection observer, the fault was reconstructed to converge to the real value of the fault [19,20,23–27].

We know that the fault may occur in the actuator of the Lur'e DI system, it is important to find a strategy to determine whether the fault would happen. If the fault occurs in the actuator, it is necessary to reconstruct the fault. However, to the authors' best knowledge, little attention has been paid to the FDI problem for the Lur'e DI system. Motivated by the previous discussion, this paper considers actuator fault detection and estimation for the Lur'e DI system. The paper is organized as follows: Section 2 presents the problem formulation and preliminaries. Section 3 designs an adaptive full-order observer to detect the actuator fault. Section 4 presents a method to estimate the actuator fault based on a reduced-order observer. Section 5 gives an example of rotor system to illustrate the effectiveness of the proposed method.

Notations: Throughout this paper, R^n denotes the n-dimensional Euclidean space, $R^{n\times m}$ is the set of $n\times m$ real matrices. $||x||$ denotes the Euclidean norm of the vector x, x^T stands for the transposition of the vector x, A^T is the transposition of the matrix A, $rank(A)$ represents the rank of the matrix A and $\lambda_{\min}(A)$ denotes the minimal eigenvalue of the matrix A. $P > (<)0$ means the positive (negative) definite matrix P with $P = P^T, I$ is the identity matrix with appropriate dimensions. $Graph(\mathcal{F})$ stands for the graph of the set-valued function $\mathcal{F}(x)$, i.e., $Graph(\mathcal{F}) = \{(x, x^*)|x^* \in \mathcal{F}(x)\}$. Absolutely continuous is shorten as AC.

2. Problem formulation and preliminaries

Let us consider the following Lur'e DI system with actuator fault:

$$\begin{cases} \dot{x} = Ax + G\omega + D\Phi(x) + Bu + Eu_f, \\ \omega \in -\rho(Hx), \\ y = Cx, \end{cases} \tag{1}$$

where $x \in R^n$ is the state, $u \in R^m$ is the control input, and $y \in R^q$ is the measurable output. $\rho : R^r \to R^r$ is a set-valued mapping, $\omega \in R^r$ is the output of ρ and stands for the multi-valued nonlinear input of the system. $\Phi : R^n \to R^p$ is a known smooth matrix function. The signal $u_f \in R^l$ represents the unknown actuator fault vector, the norm of which is bounded. $A \in R^{n\times n}, G \in R^{n\times r}, D \in R^{n\times p}, B \in R^{n\times m}, E \in R^{n\times l}, H \in R^{r\times n}$ and $C \in R^{q\times n}$ are determined matrices.

The nonlinear multi-valued term $\omega \in -\rho(Hx)$ plays an important role in practical applications when we have to adopt accurate models for the real systems. It is used to describe Coulomb friction in the rotor system, which can be seen from the Simulation part.

Firstly, we give some basic definitions of DI, detailed presentation is referred to [28].

Definition 1. [28] Let $J \subset R^m$. A set-valued mapping $\mathcal{F} : J \to R^m$ with non-empty values is said to be upper semi-continuous at $x \in J$, if for any open set U containing $\mathcal{F}(x)$, there exists an open neighborhood M of x such that $\mathcal{F}(M) \subset U$. The mapping $\mathcal{F}$ is said to be upper semi-continuous if it is upper semi-continuous at every $x \in J$.

Definition 2. [28] Let $\mathcal{F}(t, x(t))$ be a set-valued function. A function $x : [t_0, \infty) \to R^n$ is a solution to the DI $\dot{x}(t) \in \mathcal{F}(t, x(t))$ with $x(t_0) = x_0$, if $x(t)$ is AC and satisfies $\dot{x}(t) \in \mathcal{F}(t, x(t))$ for almost all $t \in [t_0, \infty)$.

Definition 3. [28] A set-valued function $\mathcal{F}(x) : R^n \to R^n$ is called monotone if its graph is monotone in the sense that for all $(x, y), (x^*, y^*) \in Graph(\mathcal{F})$ it holds that $(y - y^*)^T(x - x^*) \geqslant 0$.

In order to establish the main results of this paper, we need the following assumptions.

Assumption 1. H and C are of full row rank and E is of full column rank, i.e., $rank(H) = r < n, rank(C) = q < n$ and $rank(E) = l < n$.

Assumption 2. The set-valued mapping $\rho(\cdot)$ satisfies:

Assumption 2-1. $\rho(\cdot)$ is non-empty, convex, closed, bounded and only set-valued on a countable set of points, and is continuous on the other points.

Assumption 2-2. $\rho(\cdot)$ is monotone.

Assumption 3. The nonlinear matrix function $\Phi(x)$ is Lipschitz in x with Lipschitz constant γ, i.e.,

$$||\Phi(x) - \Phi(\hat{x})|| \leqslant \gamma ||x - \hat{x}||, \tag{2}$$

where γ is unknown.

Assumption 4. There exist matrices $P \in R^{n\times n} > 0,\ Q \in R^{n\times n} > 0,\ L \in R^{n\times q},\ F \in R^{r\times q},\ M \in R^{p\times q}$ and $N \in R^{l\times q}$ such that

$$P(A - LC) + (A - LC)^T P = -Q, \tag{3}$$

$$G^T P = H - FC, \tag{4}$$

$$D^T P = MC, \tag{5}$$

$$E^T P = NC. \tag{6}$$

Remark 1. By Definition 1, Assumption 2–1 implies that the set-valued mapping $\rho(\cdot)$ is upper semi-continuous.

Remark 2. In many practical systems, the Lipschitz constant of the nonlinear term $\Phi(x)$ is usually unknown. Thus, we suppose that γ is unknown in this paper.

Remark 3. Assumption 4 is known as dissipativity condition for the system considered in this paper, and it is also the sufficient condition for the existence of the observer.

Remark 4. (3)–(6) are a set of LMEs (linear matrix equalities), the feasible solutions of these LMEs can be computed by Scilab [29].

Then, we present some lemmas which are essential to our further investigation.

Lemma 1. [28] *Let $\mathcal{F}$ be a set-valued function, we assume that $\mathcal{F}$ is upper semi-continuous and that the image of (t, x) under $\mathcal{F}$ is closed, convex and bounded for all $t \in R$ and $x \in R^n$. Then, for each $x_0 \in R^n$ there exists an AC function $x(t)$ defined on $[0, \infty)$, which is a solution of the initial value problem $\dot{x}(t) \in \mathcal{F}(t, x(t)), x(0) = x_0$.*

Lemma 2. [30] *If $V : R \to R$ is a non-decreasing function and if $V(t) \leqslant M$ for some $M \in R$ and all $t \in R$, then $V(\cdot)$ converges.*

3. Fault detection based on full-order adaptive observer

In this section, we construct the adaptive full-order observer to detect the actuator fault. The adaptive full-order observer of the system (1) is designed as follows:

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + G\hat{\omega} + D\Phi(\hat{x}) + Bu + L(y - C\hat{x}) + \frac{1}{2}\hat{\beta}DM(y - C\hat{x}), \\ \hat{\omega} \in -\rho(H\hat{x} + F(y - C\hat{x})), \end{cases} \tag{7}$$

with the adaption law

$$\dot{\hat{\beta}} = \eta ||M(y - C\hat{x})||^2, \tag{8}$$

where η is a positive constant. The observer used in (7) is called an extended observer and was introduced by Arcak and Kokotovic [31].

Remark 5. In view of Remark 1 and Lemma 1, Assumptions 2-1 and 3 guarantee the existence of the solution of (1) and (7) when the input $u(t)$ is AC.

Subtracting (7) from (1), we obtain the following error system

$$\begin{cases} \dot{e} = (A - LC)e + G(\omega - \hat{\omega}) + D\tilde{\Phi} - \frac{1}{2}\hat{\beta}DMCe + Eu_f, \\ \omega \in -\rho(Hx), \\ \hat{\omega} \in -\rho(H\hat{x} + FCe), \\ \dot{\hat{\beta}} = \eta ||MCe||^2, \end{cases} \tag{9}$$

where $e = x - \hat{x}, \tilde{\Phi} = \Phi(x) - \Phi(\hat{x})$. Now we can state Theorem 1 of the paper.

Theorem 1. *Let $u_f = 0$ and u be an AC input. Consider the system (1), the adaptive full-order observer (7) with (8) and the error system (9). If Assumptions 1–4 hold, then (7) with (8) is an asymptotic observer of the system (1), i.e.,* $\lim_{t\to\infty} e(t) = 0$.

Proof. Let $\tilde{\beta} = \beta - \hat{\beta}$, where $\dot{\beta} = \frac{\gamma^2}{\varepsilon}$ and the constant ε satisfies $\dot{0} < \varepsilon \leqslant \frac{1}{2}\lambda_{\min}(Q)$. Then $\dot{\tilde{\beta}} = -\dot{\hat{\beta}} = -\eta||MCe||^2$. Consider the following Lyapunov function candidate

$$V = e^T Pe + \frac{1}{2}\eta^{-1}\tilde{\beta}^2. \tag{10}$$

By the fact that a quadratic function of AC functions is itself AC, since e and $\tilde{\beta}$ are AC, the derivative of V exists a.e. Taking the derivative of V along the trajectories of (9) results in

$$\begin{aligned}\dot{V} &= 2e^T P\dot{e} + \eta^{-1}\tilde{\beta}\dot{\tilde{\beta}} = 2e^T P[(A - LC)e + G(\omega - \hat{\omega}) + D\tilde{\Phi} - \frac{1}{2}\hat{\beta}DMCe] - \tilde{\beta}||MCe||^2 \\ &= 2e^T P(A - LC)e + 2e^T PG(\omega - \hat{\omega}) + 2e^T PD\tilde{\Phi} - \hat{\beta}e^T PDMCe - \tilde{\beta}||MCe||^2.\end{aligned} \tag{11}$$

By (3), then

$$2e^T P(A - LC)e = e^T[P(A - LC) + (A - LC)^T P]e = -e^T Qe. \tag{12}$$

It follows from (4) that

$$2e^T PG(\omega - \hat{\omega}) = 2e^T (H - FC)^T(\omega - \hat{\omega}) = 2[(H - FC)e]^T(\omega - \hat{\omega}),$$

where $\omega \in -\rho(Hx)$ and $\hat{\omega} \in -\rho(H\hat{x} + FCe)$. In view of the monotone property of the set-valued mapping $\rho(\cdot)$ and Definition 3, the following inequality holds

$$[(H - FC)e]^T(\omega - \hat{\omega}) = -[(H - FC)e]^T[-\omega - (-\hat{\omega})] \leq 0,$$

which means that

$$2e^T PG(\omega - \hat{\omega}) \leq 0. \tag{13}$$

By (2) and (5), we obtain that

$$2e^T PD\tilde{\Phi} = 2(MCe)^T\tilde{\Phi} \leq 2||MCe||||\tilde{\Phi}|| \leq 2\gamma||MCe||||e|| = 2||\gamma MCe||||e|| \leqslant \frac{\gamma^2}{\varepsilon}||MCe||^2 + \varepsilon||e||^2 \tag{14}$$

and

$$\hat{\beta}e^T PDMCe = \hat{\beta}e^T(MC)^T MCe = \hat{\beta}||MCe||^2. \tag{15}$$

Substituting (12)–(15) into (11) yields

$$\dot{V} \leqslant e^T(-Q + \varepsilon I)e + \left(\frac{\gamma^2}{\varepsilon} - \hat{\beta} - \tilde{\beta}\right)||MCe||^2, \tag{16}$$

which implies that

$$\dot{V} \leqslant e^T(-Q + \varepsilon I)e. \tag{17}$$

In view of the expression of ε, then $e^T(-Q + \varepsilon I)e \leqslant -\varepsilon e^T e$, which deduces to

$$\dot{V} \leqslant -\varepsilon e^T e. \tag{18}$$

Integrating both sides of (18) from 0 to t, (18) becomes

$$V(t) - V(0) \leqslant -\int_0^t \varepsilon e^T(s)e(s)ds. \tag{19}$$

In view of the fact that $V(t) > 0$ and $V(0) < \infty$, (19) implies that

$$\int_0^t \varepsilon e^T(s)e(s)ds \leqslant V(0) < \infty. \tag{20}$$

By Lemma 2, we can conclude that $\int_0^t \varepsilon e^T(s)e(s)ds$ converges, which means that $\lim_{t\to\infty} e(t) = 0$. We have completed the proof. □

Remark 6. To obtain (13), we have employed (4). This technique that allows to use the chain rule from convex analysis has been introduced by Brogliato [32].

Remark 7. If no fault occurs in the actuator, the state of the observer (7) with (8) asymptotically converges to the state of the system (1).

Remark 8. From (18), we can get the conclusion that the equilibrium $\tilde{\beta} = 0$ of the error system (9) is stable but not asymptotically stable, and $\hat{\beta}$ may not converge to the nominal value β.

Remark 9. The advantage of the designed observer is that γ is unknown and ε need not be designed. This is because γ and ε are injected into the constant $\dot{\beta} = \frac{\gamma^2}{\varepsilon}$, which can be regulated by the adaption law (8).

When a fault occurs, i.e., $u_f \neq 0$, the observer (7) with (8) is also called detection observer for the system (1). Based on the detection observer, we propose an approach for detecting the actuator fault, which is presented as follows:

$$\begin{cases} \text{if} \|e_y\| \leqslant \mu, & \text{then no fault occurs,} \\ \text{if} \|e_y\| > \mu, & \text{then a fault occurs in the actuator,} \end{cases} \tag{21}$$

where $e_y = y - C\hat{x}$ and the alarm threshold μ is a pre-specified positive constant.

4. Fault estimation based on reduced-order observer

In this section, we present a method to estimate the fault by the reduced-order observer. In order to simplify the following proof, we assume that $C = [I_q \quad 0]$.

Remark 10. From the theory of linear system, we know that there exists a linear transformation T such that $CT^{-1} = [I_q \quad 0]$. Thus the assumption that $C = [I_q \quad 0]$ is not too restrictive.

We decompose $x, A, G, D, B, E, H, P, Q$ into the forms as

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}, \quad D = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix},$$

$$E = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}, \quad H = [H_1 \quad H_2], \quad P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix}, \quad Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix},$$

where $x_1 \in R^q,\ A_{11} \in R^{q\times q},\ G_1 \in R^{q\times r},\ D_1 \in R^{q\times p},\ B_1 \in R^{q\times m},\ E_1 \in R^{q\times l},\ H_1 \in R^{r\times q},\ P_{11} \in R^{q\times q},\ Q_{11} \in R^{q\times q}$. Denote that

$$K = -P_{22}^{-1}P_{12}^T. \tag{22}$$

We have Theorem 2 which is necessary for the following discussion.

Theorem 2. *Let K be defined in (22). If Assumption 4 holds, then*

$$P_{22}(A_{22} - KA_{12}) + (A_{22} - KA_{12})^T P_{22} = -Q_{22}, \tag{23}$$

$$(G_2 - KG_1)^T P_{22} = H_2, \tag{24}$$

$$-KD_1 + D_2 = 0, \tag{25}$$

$$-KE_1 + E_2 = 0. \tag{26}$$

Proof. Since $C = [I_q \quad 0]$, by the property of block matrix, (3)–(6) leads to

$$P_{12}^T A_{12} + A_{12}^T P_{12} + P_{22}A_{22} + A_{22}^T P_{22} = -Q_{22}, \tag{27}$$

$$G_1^T P_{12} + G_2^T P_{22} = H_2, \tag{28}$$

$$D_1^T P_{12} + D_2^T P_{22} = 0, \tag{29}$$

$$E_1^T P_{12} + E_2^T P_{22} = 0. \tag{30}$$

Substituting (22) into (27)–(30) and considering the fact that $P_{22} > 0$ is inversible, we can complete the proof of Theorem 2. $\square$

Remark 11. The step of computing K is as follows: Firstly, we use Scilab to solve the matrix P from (3)–(6). Then, we use the decomposition of P, i.e., $\dot{P} = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix}$ to compute K.

Now we design the reduced-order observer for the system (1) as

$$\begin{cases} \dot{\hat{z}} = (A_{22} - KA_{12})\hat{z} + (G_2 - KG_1)\hat{\omega} + (B_2 - KB_1)u + [(A_{21} - KA_{11}) + (A_{22} - KA_{12})K]y, \\ \hat{\omega} \in -\rho(H_2\hat{z} + (H_1 + H_2K)y), \\ \hat{x}_2 = \hat{z} + Ky. \end{cases} \tag{31}$$

Theorem 3. *Let u be an AC input and K be given in (22). If Assumptions 1–4 hold, then (31) is a reduced-order observer of the system (1), i.e.,* $\lim_{t\to\infty}[x_2(t) - \hat{x}_2(t)] = 0$.

Proof. Since $y = x_1$, we can write the system (1) as

$$\begin{cases} \dot{y} = A_{11}y + A_{12}x_2 + G_1\omega + D_1\Phi(x) + B_1u + E_1u_f, \\ \dot{x}_2 = A_{21}y + A_{22}x_2 + G_2\omega + D_2\Phi(x) + B_2u + E_2u_f, \\ \omega \in -\rho(H_1y + H_2x_2). \end{cases} \tag{32}$$

Denote that $z = x_2 - Ky$, then

$$\begin{cases} \dot{z} = (A_{22} - KA_{12})z + (G_2 - KG_1)\omega + (D_2 - KD_1)\Phi(x) + (B_2 - KB_1)u + (E_2 - KE_1)u_f + [(A_{21} - KA_{11}) + (A_{22} - KA_{12})K]y, \\ \omega \in -\rho(H_2z + (H_1 + H_2K)y), \\ x_2 = z + Ky. \end{cases} \tag{33}$$

In view of (25) and (26), (33) becomes

$$\begin{cases} \dot{z} = (A_{22} - KA_{12})z + (G_2 - KG_1)\omega + (B_2 - KB_1)u + [(A_{21} - KA_{11}) + (A_{22} - KA_{12})K]y, \\ \omega \in -\rho(H_2z + (H_1 + H_2K)y), \\ x_2 = z + Ky. \end{cases} \tag{34}$$

Subtracting (31) from (34) yields the following error system

$$\begin{cases} \dot{e}_z = (A_{22} - KA_{12})e_z + (G_2 - KG_1)(\omega - \hat{\omega}), \\ \omega \in -\rho(H_2z + (H_1 + H_2K)y), \\ \hat{\omega} \in -\rho(H_2\hat{z} + (H_1 + H_2K)y), \end{cases} \tag{35}$$

where $e_z = z - \hat{z}$. Choosing the Lyapunov function candidate as $V(e_z) = e_z^TP_{22}e_z$, we have

$$\dot{V}(e_z) = 2e_z^TP_{22}\dot{e}_z = e_z^T[P_{22}(A_{22} - KA_{12}) + (A_{22} - KA_{12})^TP_{22}]e_z + 2e_z^TP_{22}(G_2 - KG_1)(\omega - \hat{\omega}). \tag{36}$$

By (23), then

$$e_z^T[P_{22}(A_{22} - KA_{12}) + (A_{22} - KA_{12})^TP_{22}]e_z = -e_z^TQ_{22}e_z. \tag{37}$$

Considering (24), the following equality holds

$$2e_z^TP_{22}(G_2 - KG_1)(\omega - \hat{\omega}) = 2e_z^TH_2^T(\omega - \hat{\omega}),$$

where $\omega \in -\rho(H_2z + (H_1 + H_2K)y)$ and $\hat{\omega} \in -\rho(H_2\hat{z} + (H_1 + H_2K)y)$. By the monotone property of the set-valued mapping $\rho(\cdot)$, we have

$$e_z^TH_2^T(\omega - \hat{\omega}) = -(H_2e_z)^T[-\omega - (-\hat{\omega})] \le 0,$$

which means that

$$2e_z^TP_{22}(G_2 - KG_1)(\omega - \hat{\omega}) \le 0. \tag{38}$$

Substituting (37), (38) into (36) results in

$$\dot{V}(e_z) \leqslant -e_z^TQ_{22}e_z. \tag{39}$$

Since $Q_{22} > 0$, (39) implies that $\dot{V}(e_z) < 0$. Thus we can conclude that $\lim_{t\to\infty}[\hat{z}(t) - z(t)] = 0$, i.e., $\lim_{t\to\infty}[\hat{x}_2(t) - x_2(t)] = 0$. □

We now use the reduced-order observer (31) to estimate the actuator fault u_f of the system (1), which is given as follows:

$$\hat{u}_f = (E^TE)^{-1}E^T[\dot{\hat{x}} - A\hat{x} - G\hat{\omega} - D\Phi(\hat{x}) - Bu], \tag{40}$$

where $\hat{x} = [y^T, \hat{x}_2^T]^T$, and $\hat{x}_2$ is generated by the reduced-order observer (31).

Remark 12. Theorem 3 shows that $\hat{x}_2$ always converges to x_2 asymptotically whether $u_f = 0$ or not, thus the performance of the reduced-order observer (31) is not affected by the fault u_f. Because (26) decouples u_f from the error dynamics (hence so does (6)), (5) and (6) are the big hypothesis which make the reduced-order observer work.

Remark 13. In real systems, $\dot{\hat{x}}$ is difficult to compute due to the presence of the noise. We can resort to the method introduced by Dierckx [33] to treat with the derivative of $\hat{x}$.

5. Simulation

In this section, we take the rotor system in [4] for example. The rotor system is defined as: The input of rotor system is the voltage, which is between 5 and 5 V. The upper and lower discs are connected through a low-stiffness steel string, and the dc-motor is connected to the upper steel disc by the gear box. Both discs can rotate around their geometric centers and the related angular positions are measured by incremental encoders. And a brake apparatus is installed at the lower disc and creates a friction that induces limit cycling to the system.

Let us consider the system (1) with

$$A = \begin{bmatrix} 0 & 1 & -1 \\ -0.1526 & -4.6688 & 0 \\ 2.2301 & 0 & 0.6442 \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ 0 \\ 30.6748 \end{bmatrix},$$

$$D = \begin{bmatrix} 8.0287 \\ -11.7230 \\ -14.7670 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 8.3841 \\ 0 \end{bmatrix}, \quad E = \begin{bmatrix} 9.6344 \\ -14.0676 \\ -17.7204 \end{bmatrix},$$

$$H = [0 \quad 0 \quad 1], \quad C = [1 \quad 0 \quad 0], \quad \Phi(x) = 2\sin x_2, \quad u = 2.$$

The set-valued mapping $\rho(\lambda)$ is the Coulomb friction and given by

$$\rho(\lambda) = \begin{cases} [0.1642 + 0.0603(1 - \frac{2}{1+e^{5.7468|\lambda|}}) \\ \quad -0.2267(1 - \frac{2}{1+e^{0.2941|\lambda|}})]sign(\lambda) + 0.0319\lambda, & \lambda \neq 0, \\ [-0.1642, 0.1642], & \lambda = 0. \end{cases}$$

The figure of $\rho(\lambda)$ is shown in Fig. 1.

It is known from [4] that when $\Phi(x) = u_f = 0$, the model stands for the rotor system (after loop transformation). In the model, the states are defined as

$$x_1 = \theta_u - \theta_l, \quad x_2 = \dot{\theta}_u, \quad x_3 = \dot{\theta}_l,$$

where θ_u and θ_l are the angular positions of the upper and lower discs, respectively, and u is the input voltage.

In order to show that our design method is effective for the Lur'e DI system, we add $\Phi(x)$ and u_f to the original model. Solving the LMEs (3)–(6) yields

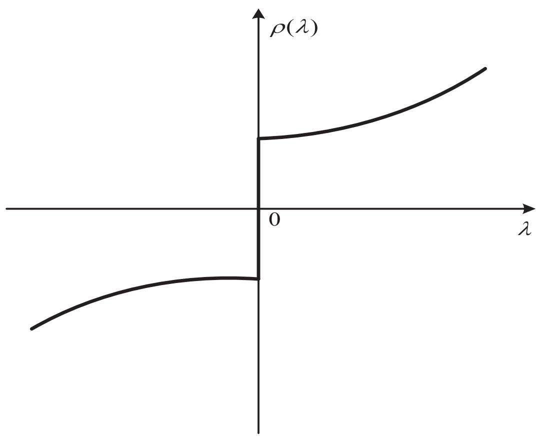

Fig. 1. The figure of the friction law $\rho(\lambda)$.

$$P=\begin{bmatrix}0.6745 & 0.1052 & 0.0600\\ 0.1052 & 0.0721 & 0\\ 0.0600 & 0 & 0.0326\end{bmatrix},\quad Q=\begin{bmatrix}0.4346 & 0 & 0\\ 0 & 0.4624 & 0.0452\\ 0 & 0.0452 & 0.0779\end{bmatrix},$$

$$L=\begin{bmatrix}2.7277\\ -1.5888\\ -22.2963\end{bmatrix},\quad F=-1.8393,\quad M=3.2969,\quad N=3.9562.$$

Firstly, we present the adaptive full-order observer to detect the fault. The adaptive full-order observer for the system (1) is

$$\begin{cases}\dot{\hat{x}}_1=\hat{x}_2-\hat{x}_3+8.0287\Phi(\hat{x}_2)+2.7280(y-\hat{x}_1)+13.2349\hat{\beta}(y-x_1),\\ \dot{\hat{x}}_2=-0.1526\hat{x}_1-4.6688\hat{x}_2-11.7230\Phi(\hat{x}_2)\\ \qquad +8.3841u-1.5898(y-\hat{x}_1)-19.3248\hat{\beta}(y-x_1),\\ \dot{\hat{x}}_3=2.2301\hat{x}_1+0.6442\hat{x}_3+30.6748\hat{\omega}-14.7670\Phi(\hat{x}_2)\\ \qquad -22.2970(y-\hat{x}_1)-24.3427\hat{\beta}(y-x_1),\\ \hat{\omega}\in-\rho(\hat{x}_3-1.8393(y-\hat{x}_1)),\\ \dot{\hat{\beta}}=0.01[3.2969(y-\hat{x}_1)]^2.\end{cases} \tag{41}$$

The initial states of the systems (1) and (41) are chosen as $[0\quad 0\quad 0]^T$ and $[1\quad 0.1\quad 0.3]^T$, and the initial value of $\hat{\beta}$ is 1. When $u_f=0$, Fig. 2 shows that the errors e_1, e_2 and e_3 of the error system (9) converge to zero asymptotically. When $u_f\neq 0$ and is defined as

$$u_f=\begin{cases}5, & 20\leqslant t<30,\\ 6, & 40\leqslant t<50,\\ 5.5, & 80\leqslant t<90,\\ 0, & \text{else}.\end{cases}$$

We detect the occurrence of the fault by (21), where the alarm threshold $\mu=0.3$. From Fig. 3, we see that the alarm device works during the time intervals $[20,30]$, $[40,50]$ and $[80,90]$, which is in accordance with the expression of u_f.

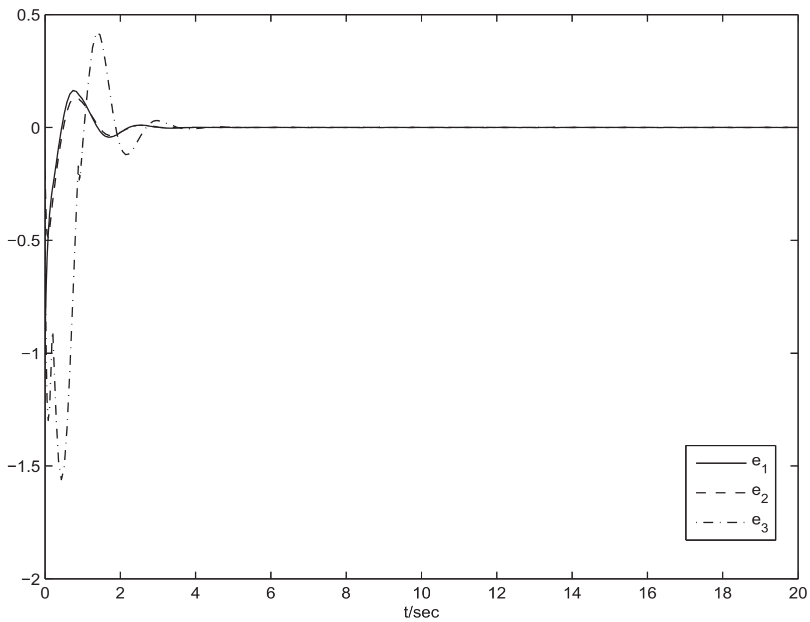

Fig. 2. The response of the errors e_1, e_2 and e_3 of the full-order error system when no fault occurs.

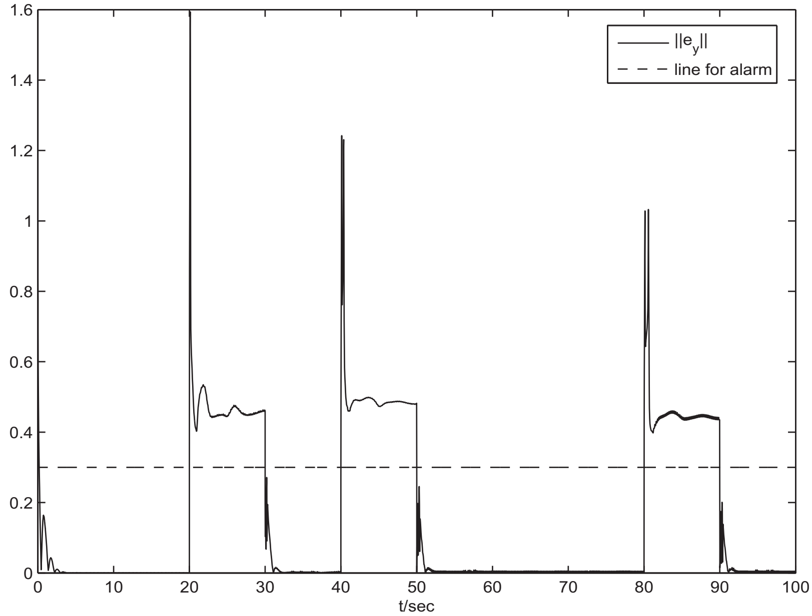

Fig. 3. The response of $||e_y||$ and the line for alarm.

Secondly, we design the reduced-order observer and estimate the fault. The reduced-order observer for the system (1) is

$$\begin{cases} \dot{\hat{z}}_1 = -3.2097\hat{z}_1 - 1.4591\hat{z}_2 + 8.3841u + 7.2161y, \\ \dot{\hat{z}}_2 = -1.8405\hat{z}_1 - 1.1963\hat{z}_2 + 30.6748\hat{\omega} + 1.7464y, \\ \hat{\omega} \in -\rho(\hat{z}_2 - 1.8405y), \\ \hat{x}_2 = \hat{z}_1 - 1.4591y, \\ \hat{x}_3 = \hat{z}_2 - 1.8405y. \end{cases} \tag{42}$$

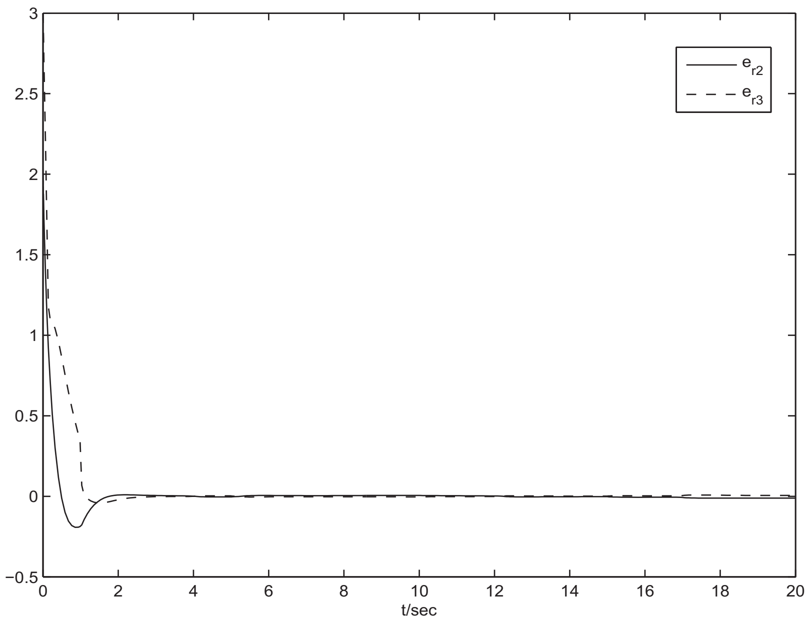

Fig. 4. The response of the errors e_{r2} and e_{r3}.

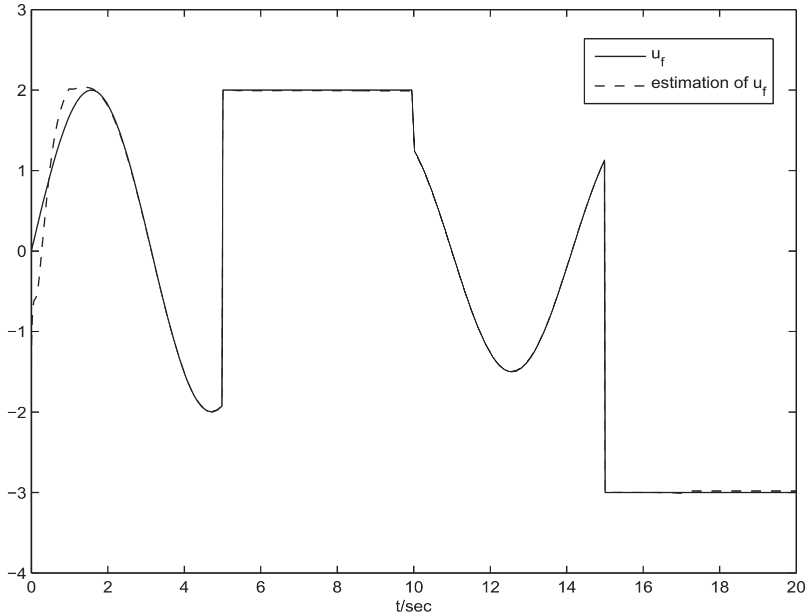

Fig. 5. The response of the fault u_f and the estimation $\hat{u}_f$.

The fault u_f in system (1) is chosen as

$$u_f = \begin{cases} 2\sin t, & 0 \leqslant t < 5, \\ 2, & 5 \leqslant t < 10, \\ -1.5\cos t, & 10 \leqslant t < 15, \\ -3, & \text{else}. \end{cases}$$

The initial state of (42) is $[\hat{z}_1(0), \hat{z}_2(0)] = [2, 3]^T$. Denote that $e_{r2} = \hat{x}_2 - x_2$ and $e_{r3} = \hat{x}_3 - x_3$. Fig. 4 shows the reduced-order observer works well. Then, using (40), we reconstruct $u_f(t)$, which is shown in Fig. 5. From the simulation results, we conclude that our design method is valid.

6. Conclusion

We consider the actuator fault detection and estimation for the Lur'e DI system. Firstly, we use an adaptive full-order observer to detect the actuator fault. Then, we reconstruct the actuator fault by a reduced-order observer. Finally, we give an example of rotor system to show the effectiveness of the design method.

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