

## Robust Beamformer in the Presence of Mutual Coupling

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### Abstract-

The performance of antenna array processors badly suffers in the presence of mutual coupling (MC). The mutual coupling between the array sensors can significantly change the communication characteristics of an antenna array. In this paper, we show the effect of mutual coupling on the radiation pattern as well as output SINR for conventional and optimal beamforming. To minimize this mutual coupling effect on the antenna array, we propose a robust beamforming technique named Fixed Diagonal Loading (FDL). Simulation results show the effectiveness of the proposed technique in the presence of very strong mutual coupling effect.

### Keywords—

Smart Antenna System, Beamforming, Mutual Coupling, Fixed Diagonal Loading, Signal to Interference plus Noise Ratio (SINR).

### 1.INTRODUCTION

Antenna array signal processing plays an important role in many applications involving communication, sonar, radar, acoustics and many more[1-3]. One of the important tasks of antenna array processing is beamforming. The standard beamformers include the delay-and-sum approach, which is known to suffer from poor resolution and high side lobe problems. The Capon beamformer adaptively selects the weight vectors to minimize the array output power subject to the linear constraint that the signal of interest (SOI) does not suffer from any distortion [4]. There is a tendency to develop more compact antenna arrays with a larger number of antenna elements. Because of this the antennas are located closer to each other and the effects of mutual coupling become more significant. Mutual coupling is due to the fact that when an antenna is radiating, some of the energy in one antenna is coupled into a neighboring antenna [6]. In combination with scattering effects from the antenna tower and nearby structures the radiation characteristics of an antenna in such an antenna array can differ significantly from the stand-alone antenna characteristics [6-8].

Antenna array processors suffer from performance deterioration in the presence of mutual coupling (MC) between array sensors. It has been pointed out that the performance of an adaptive array antenna is affected by the mutual coupling and this effect, which is particularly serious for small array spacing, decreases the output signal-to-interference-noise ratio (SINR) and degrades the convergence of the least mean square (LMS) [5]. Output SINR is increased by the gain obtained in the adaptive processing based on the input SINR, both the output SINR and the input SINR were investigated in the presence of the mutual coupling to understand which is fundamentally affected by the mutual coupling[7]. A radiation pattern for a given set of properties, such as main beam direction and gain, position and level of nulls maximum sidelobe level in a specified region are also affected by MC effect which is addressed in [9].

In the past three decades many approaches have been proposed to improve the robustness of the beamforming in the presence of mutual coupling. These approaches include the universal steering vector method (USV) [7], optimal beamforming algorithm to rely on the active element pattern instead of compensating MC effect [8]. Additional linear constraints, including point and derivative constraints, have been imposed to improve the robustness of the capon beamforming [4]. To obtain an appropriate weight vector for optimum narrow band beamformer without MC effect, minimization of mean square error (MSE) and maximum likelihood estimation (MLE) are used [10]. Constrained Particle Swarm Optimization (PSO) method is used for solving constrained optimization problem encountered in beamforming systems [9].

To compensate the MC effect on antenna array locating close to each other, this paper proposed a robust antenna array system in the presence of strong mutual coupling effects which will be computationally more efficient than the existing beamformer. In this paper, the output signal to interference plus noise ratio (SINR) is increased by the gain obtained in the adaptive processing based on the input SINR, the output SINR is investigated for various parameter variation such as antenna array spacing, noise level, beam-pointing direction, number of interferences in the presence of mutual coupling to understand which parameter fundamentally affects the MC. However constructing the MC matrix remains a crucial problem. We build the MC matrix in a smart antenna system [6]. FDL method is a method that adds a constant to the diagonal of the covariance matrix. This robust technique will enhance the SINR and decrease the sidelobe level in the presence of MC effects.

This paper is organized as follows. Section 2 describes the modeling of signal, array geometry and the background knowledge for this paper. Section 3 presents different beamforming technique including our proposed robust with the presence of MC effects. In section 4, we consider the performance evaluation for existing methods and our proposed robust method both for without and with the presence of MC. Lastly, A conclusion is drawn in section 5.

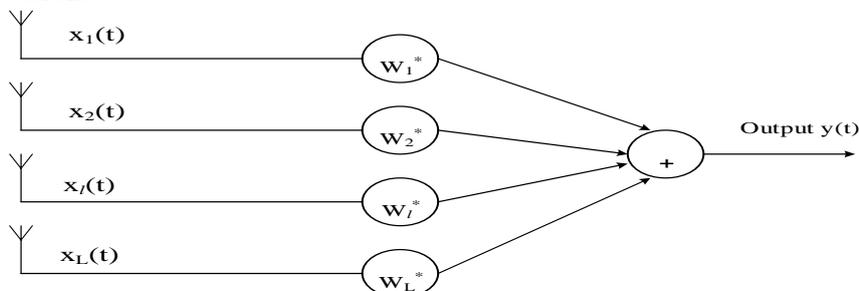
## 2. SIGNAL MODEL

### 2.1. ARRAY SYSTEM:

Consider the antenna array system consisting of L antenna elements shown in Fig.1, where signals from each element are multiplied by a complex weight and summed to form the array output. The figure does not show components such as preamplifiers, bandpass filters, and so on. It follows from the figure that an expression for the array output is given by [1]

$$y(t) = \sum_1^L W_1^* X_1(t) \tag{1}$$

Where\* denotes the complex conjugate. The conjugate of complex weights is used to simplify the mathematical notation.



**Fig.1. Antenna array system.**

Denoting the weights of the array system using vector notation as  $W = [W_1, W_2, \dots, W_L]^T$  (2)

and signals induced on all elements as

$$x(t) = [x_1(t), x_2(t), \dots, x_L(t)]^T \tag{3}$$

The output of the array system becomes

$$y(t) = W^H x(t) \tag{4}$$

Where superscript T and H, respectively, denote transposition and the complex conjugate transposition of a vector or matrix. Throughout the paper  $w$  and  $x(t)$  are referred to as the weight vector and the signal vector, respectively. Note that to obtain the array output, you need to multiply the signals induced on all elements with the corresponding weights. In vector notation, this operation is carried out by taking the inner product of the weight vector with the signal vector.

The output power of the array at any time  $t$  is given by the magnitude square of the array output, that is,

$$\begin{aligned} P(t) &= |y(t)|^2 \\ &= y(t)y^*(t) \end{aligned} \tag{5}$$

Substituting for  $y(t)$  from (4), the output power becomes

$$P(t) = W^H x(t)x^H(t)W \tag{6}$$

If the components of  $x(t)$  can be modeled as zero-mean stationary processes, then for a given  $w$  the mean output power of the array system is obtained by taking conditional expectation over  $x(t)$ :

$$\begin{aligned} P(W) &= E[W^H x(t)x^H(t)W] \\ &= W^H E[x(t)x^H(t)]W \\ &= W^H RW \end{aligned} \tag{7}$$

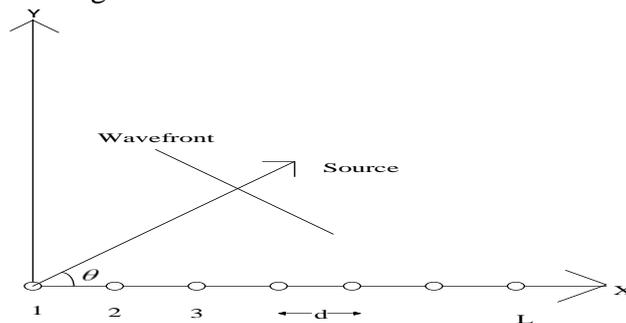
Where  $E[\cdot]$  denotes the expectation operator and  $R$  is the array correlation matrix defined by

$$R = E[x(t)x^H(t)] \tag{8}$$

Elements of this matrix denote the correlation between various elements. For example,  $R_{ij}$  denotes the correlation between  $i^{\text{th}}$  and the  $j^{\text{th}}$  element of the array.

### 2.2. Array geometry:

The most commonly used array geometries are 1. Linear array (LA), 2. Planar array, 3. Volumetric array. In linear array configuration array sensors can be located arbitrarily on a line. Linear array system can be categorized into two types namely Uniform Linear Array (ULA) and Non-Uniform Linear Array. In ULA, the distance between array sensors are uniform with uniform set of antenna elements. But in Non-Uniform Linear Array, the antenna elements may be of different kinds or have different properties. For this, each array of elements has different antenna patterns. The array geometry for  $L$  element ULA is given below.



**Fig.2. Uniform Linear Array with element spacing  $d$**



In fig. 2. We have considered an L element ULA with element spacing d. We have also assumed that the array elements are aligned with the x-axis such that the first element is situated at the origin. The time taken by a plane wave arriving from  $k^{\text{th}}$  source and measured from  $l^{\text{th}}$  element is

$$\tau_l = \frac{d}{c} (L-1) \cos \theta \quad (9)$$

Let  $S_k$  denote the steering vector associated with the  $k^{\text{th}}$  source. For an array of identical elements, it is defined as

$$S_k = [\exp(j2\pi f_0 \tau_1(\phi_k, \theta_k)), \dots, \exp(j2\pi f_0 \tau_L(\phi_k, \theta_k))]^T \quad (10)$$

Note that when the first element of the array is at the origin of the coordinate system  $\tau_1(\phi_k, \theta_k) = 0$ , the first element of the steering vector is identical to unity [1].

### 3. BEAMFORMING TECHNIQUE

A phased array antenna uses an array of antennas. Each antenna forming the array is known as an element of the array. The signals induced on different elements of an array are combined to form a single output of the array. This process of combining the signals from different elements is known as beamforming. Beamforming technique is mainly categorized into two types as Conventional and Optimal beamforming technique.

#### 3.1. Conventional Beamformer:

The conventional beamformer, sometimes also known as the delay-and-sum beamformer, has weights of equal magnitudes. The phases are selected to steer the array in a particular direction  $(\phi_0, \theta_0)$  known as look direction. With  $S_0$  denoting the steering vector in the look direction, the array weights are given by [1]

$$W_c = \frac{1}{L} S_0 \quad (11)$$

The response of a processor in a direction  $(\phi, \theta)$  is obtained by using (4) that is, taking the dot product of the weight vector with the steering vector  $S(\phi, \theta)$ . With the weights given by (10) the response  $y(\phi, \theta)$  is given by

$$\begin{aligned} y(\phi, \theta) &= W_c^H S(\phi, \theta) \\ &= \frac{1}{L} S_0^H(\phi, \theta) \end{aligned} \quad (12)$$

#### 3.2. Optimal Beamformer:

The conventional scheme described in the previous section requires knowledge of the directions of interference sources, and the beamformer using the weights estimated by this scheme does not maximize the output SNR. The optimal beamforming method described in this section overcomes these limitations and maximizes the output SNR in the absence of errors. It should be noted that the optimal beamformer, also known as the minimum variance distortionless response (MVDR) beamformer, described in this section does not require knowledge of directions and power levels of interferences as well as the level of the background noise power to maximize the output SNR. It only requires the direction of the desired signal [1-3].

We now discuss an optimal beamformer with its weights with constraints.

Let the array weights be constrained to have a unit response in the look direction, that is,

$$\hat{W}^H S_0 = 1 \quad (13)$$

Thus it follows from unconstrained beamformer that constant  $\mu_0$  is given by

$$\mu_o = \frac{1}{S_o^H R_N^{-1} S_o} \quad (14)$$

Substituting this for unconstrained beamformer results in the expression for weight vector

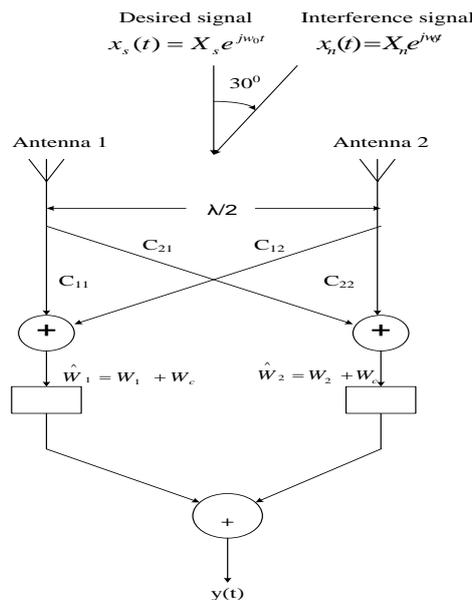
$$\hat{W} = \frac{R_N^{-1} S_o}{S_o^H R_N^{-1} S_o} \quad (15)$$

In practice when the estimate of noise alone matrix is not available, the total array correlation matrix (signal plus noise) is to estimate the weights and the processor is referred to as the SPNMI (signal plus noise matrix inverse). An expression for the weights of the constrained processor for the case is given by [1]

$$\hat{W} = \frac{R^{-1} S_o}{S_o^H R^{-1} S_o} \quad (16)$$

### 3.3. Mutual Coupling Formation:

To form the mutual coupling matrix, let us consider the two element linear array, with half-wavelength spacing, receiving a desired signal at  $\theta_0 = 0^\circ$  while tuning out an interferer (SNOI) at  $\theta_1 = 30^\circ$  in the presence of mutual coupling. The elements of the array in Fig 2. are assumed to be, for simplicity, isotropic and the impinging signals are sinusoids.



**Fig.3. MC between two array sensors**

In order to assess the effect of mutual coupling on the proposed beamformer performance, we incorporate a mutual coupling (MC) matrix in the model for the received signal, modifying array correlation matrix to

$$R = E[X_c(t)X_c^H(t)] \quad (17)$$

Where  $X_c(t) = CX(t)$  and  $C$  is a mutual coupling matrix is given by [2, 6]

$$C = [Z_A + Z_L](Z_C + Z_L I_N)^{-1} \quad (18)$$

Where  $Z_A$  is the sensor impedance without mutual coupling,  $Z_L$  is the impedance of the receiver at each sensor which is taken to be  $50\Omega$  and  $I_N$  is the identity matrix. Considering the case of an antenna array with the side-by-side configuration the mutual impedance matrix  $Z_C$  is given by [2, 6]

$$Z_C = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1M} \\ Z_{21} & Z_{22} & \dots & Z_{2M} \\ \vdots & \vdots & \vdots & \vdots \\ Z_{M1} & Z_{M2} & \dots & Z_{MM} \end{bmatrix} \quad (19)$$

Where the entry  $Z_{m,n}, m, n = 1, 2, \dots, M$  is given by

$$Z_{mn} = \begin{cases} 30[0.5772 + \ln(2\kappa\gamma) - Ci(2\kappa\gamma)] + \\ j[30(Si(2\kappa\gamma))] = Z_A \quad \text{for } m=n \\ \\ 30[2Ci(\mu_o) - Ci(\mu_1) - Ci(\mu_2)] - j \\ [30(2Si(\mu_o) - Si(\mu_1) - Si(\mu_2))] \quad \text{for } m \neq n \end{cases} \quad (20)$$

### 3.4. Proposed Robust Technique:

Adaptive beamforming optimizes a collection of weight vectors to localize targets via correlation with the data in a noisy environment. These weight vectors generate a beam-pattern that places nulls in the direction of unwanted noise (i.e., signals, called interference, from directions other than the direction of interest). In contrast to conventional beamforming where the weight vector is a constant and independent of incoming data. Minimum variance distortion less response is an adaptive algorithm which will minimize the output in all directions subject to the condition that gain in the steering direction is unity. The steering direction is the bearing that the array is steered towards to look for a particular incoming signal. This algorithm gives optimum performance by steering nulls in the direction of interference and also offers better performance in the case of correlated noise sources. The perfect MVDR beamformer is not a robust method due to a variety of reasons. One of the principal reasons for this is the neglect of Mutual Coupling between array sensors.

A common way to increase robustness of the MVDR beam former is to add a constant  $\lambda$  to the diagonal of the covariance matrix generally known as fixed diagonal loading (FDL).

$$R_{new} = R + \lambda I \quad (21)$$

The following algorithm is followed to improve the robustness of MVDR beamformer

1. Calculate steering vector  $S$  using (10)
2. Obtain array correlation matrix  $R$  from (8)
3. Update  $R$  obtaining from step 2 using (21)
4. Calculate weight vector  $W$  using (16)
5. Obtain output power using (7)

Adding a constant to the diagonal of the covariance matrix can be seen as increasing the noise level in the recorded data before finding the optimal aperture shading, assuming the noise is white. As white noise becomes dominant, the minimum variance solution approaches conventional beam forming with uniform shading. The selection of a fixed loading factor  $\lambda$  is a challenging problem. One way to determine  $\lambda$  is to set it to a fixed value with reference to the background noise or equal to the standard deviation of the diagonal entries of the correlation matrix as discussed in [2]. However, it is not clear how to relate the parameters of the background noise gain constraint and the level of uncertainty of the signal look direction. Furthermore, the relationship between the diagonal loading factor and the parameters of the background noise gain constraint is not simple, and to satisfy this constraint, a multistep iterative procedure is required to adjust the diagonal loading factor [2]. Each step of this iterative procedure involves an update of the inverse of the diagonally loaded covariance matrix, and as a result, the total computational complexity of adaptive beamforming with the

background noise gain constraint may be higher than that of the sample matrix inversion algorithm. Because of this, the diagonal loading factor is usually chosen in a more adhoc way, typically about  $10 \sigma_n^2$ , where  $\sigma_n^2$  is the background noise power at a single sensor.

#### 4. PERFORMANCE EVALUATION

To demonstrate the effect of MC on beamforming for the conventional and optimal beamforming, we consider a 10 element ULA with half wavelength spacing, Input signal power unity, three interferences with interference power 10 times of signal power, noise level of -10dB. We want to steer the beam at a direction of 100 degree azimuth and the interferences are in the position of 20°, 60°, and 140° in azimuth. In this section the performance of conventional beamforming is compared with optimal beamforming in the absence of MC as well as in the presence of MC. The robustness of proposed beamformer is evaluated here in presence of MC.

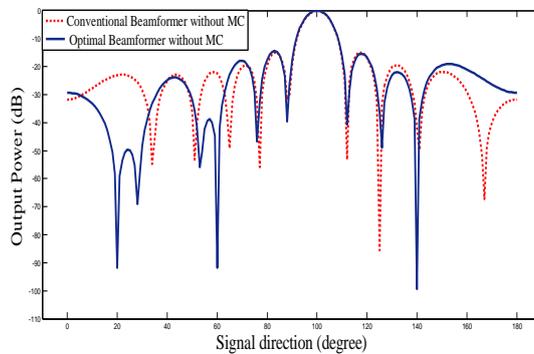


Fig. 4. Power pattern versus azimuth angle using conventional and optimal beamforming method without MC for number of elements=10 with half wavelength spacing, number of interference=3, each interference power=10.0, interference direction= [20°, 60°, 140°], signal power=1.0, signal direction=100°, background noise=0.1.

Fig. 4 shows the power pattern for both conventional and optimal beamforming in the absence of mutual coupling. It is found that the optimal beamforming gives nulls at interferences direction that means it can easily cancel the interferences but conventional beamforming is unable to cancel interferences. In the absence of MC, the optimal beamforming, the pattern null is at  $\phi=140^\circ$  at a power level of nearly -100dB which is not possible for conventional beamforming. So it can be said that the optimal beamforming gives better interferences cancellation than the conventional beamforming.

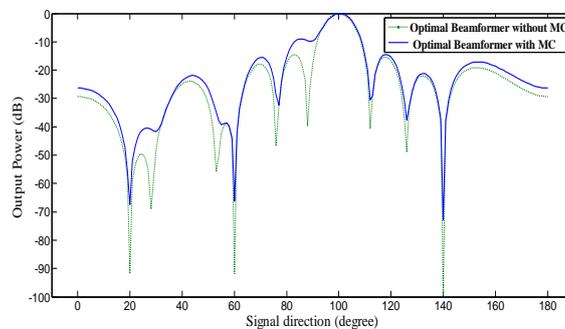


Fig. 5. Power pattern using optimal beamforming without and with mutual coupling for number of elements=10 with half wavelength spacing, number of interference=3, each interference power=10.0, interference direction= [20°, 60°, 140°], signal power=1.0, signal direction=100°, background noise=0.1.

Fig. 5. Shows the radiation pattern of the optimal beamformer in the absence and presence of mutual coupling as a function of azimuth angle. It is observed that for optimal beamformer with MC the sidelobe level has been increased significantly, the beamwidth (BW) has become small, the first null beamwidth (FNBW) has become large, and the interferences cancellation capabilities have been decreased compared with the absence of MC.

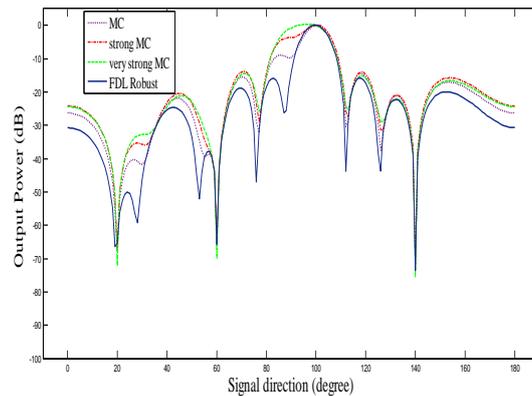


Fig. 6. Power pattern of the optimal beamformer for three different MC matrices and the proposed FDL robust technique in the presence of very strong MC.

Fig. 6 compares the power pattern using optimal beamforming with three different mutual coupling models. The beamformer is designed for same configuration of Fig. 4 with matrix  $Z_c$  is calculated by using equation (18). To analysis the effects of varying degrees of mutual coupling in the proposed beamformer, the off diagonal element of  $Z_c$  matrix are arbitrarily multiplied by 2 and 5 respectively which simulates strong and very strong mutual coupling. It is noticed that the main beam shape and beam pointing direction are unaffected by mutual coupling whereas the sidelobe level is increasing with the stronger mutual coupling effect. This figure shows that the sidelobe level becomes smaller when we apply our proposed robust technique for compensating mutual coupling. This tells us that the proposed technique is robust.

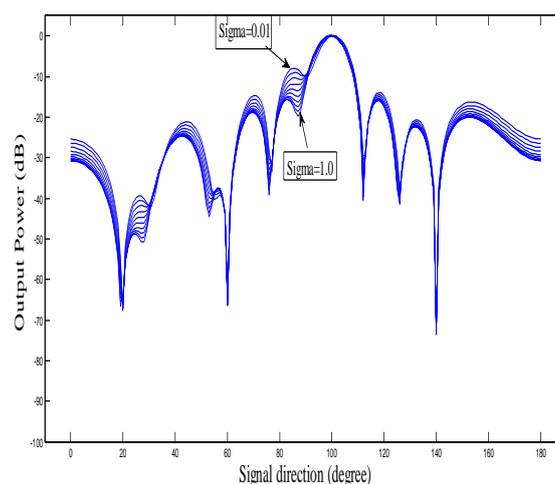


Fig. 7. Radiation pattern of optimal beamformer with mutual coupling as a function of noise power ( $\sigma$ ) for number of elements=10, number of interference=3, each interference power=10.0, interference direction=  $[20^\circ, 60^\circ, 140^\circ]$ , signal power=1.0, signal direction= $100^\circ$ .

Fig. 7. Shows the radiation pattern of optimal beamformer in the presence of mutual coupling for different noise level. It shows that the main beam shape, beam pointing direction are unaffected by

mutual coupling but the sidelobe level is increasing with the decreasing of noise level ( $\sigma$ ). This figure shows how mutual coupling effect depends on noise level.

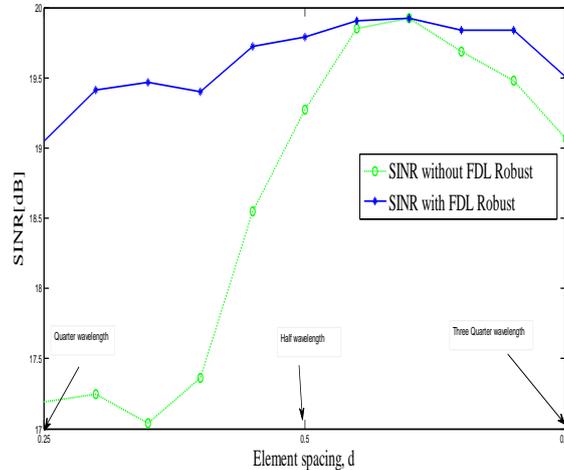


Fig. 8. Output SINR as a function of spacing for optimal beamformer with and without FDL robust beamformer for number of elements=10, number of interference=3, each interference power=10.0, interference direction= [20°, 60°, 140°], signal power=1.0, signal direction=100°, background noise=0.1.

In fig. 8, it is found that both output SINR for optimal beamformer with and without FDL robust decrease when the element spacing becomes small. For quarter wavelength spacing SINR without Robust= 17.20 dB and with Robust= 19.05dB, for half wavelength spacing without Robust= 19.25 dB and SINR with Robust= 19.80 dB and three quarter wavelength spacing SINR without Robust= 19.00 dB and with Robust= 19.50 dB. So, the proposed FDL technique maintains system SINR with Robustness.

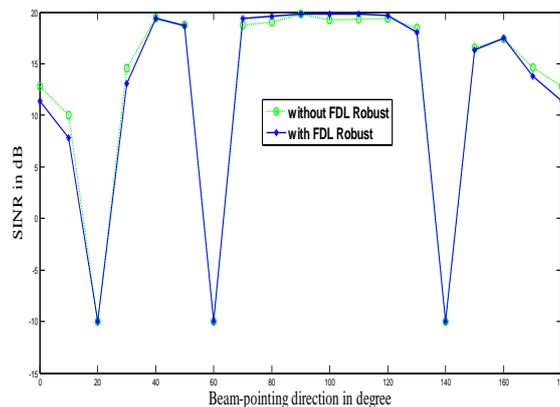


Fig. 9. Realized SINR at the output for optimal and FDL robust for a 10 element ULA with half wavelength spacing, 3 interferences with each power 10 and at position [20°, 60°, 140°], source power 1.0, noise power 0.1.

Fig. 9 presents the effect of beam pointing direction to the output SINR. Here, the interferences position are [20°, 60°, 140] in azimuth. It is found that when the beam-pointing direction is close to interference the SINR drops with a significant value. At the interferences position the array provides an SINR of -10 dB and a flat region of nearly 20 dB between the angle differences of two adjacent interferences. The optimal as well as the proposed FDL robust both the technique work almost same in all the interval.

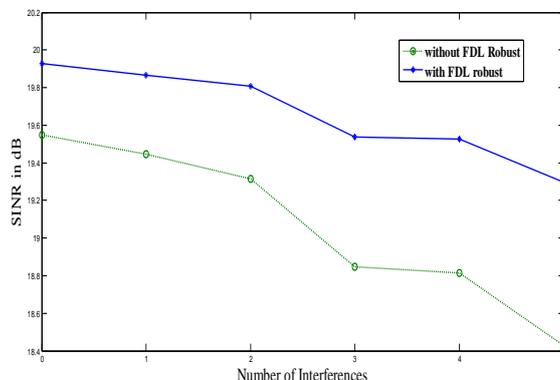


Fig. 10. Effect of number of interferences on signal output SINR of optimal beamformer without and with proposed FDL robust beamformer in the presence of MC for number of elements=10, each interference power=10.0, interferences direction [20, 60, 110, 140,160] signal power=1.0, signal direction=100°, background noise=0.1

Fig. 10. Compares the output SINR of optimal beamformer and FDL robust beamformer in the presence of MC for the variation of interferences number. It is found that the SINR is decreasing for both of the beamformer in the increase of number of interferences. Both gives highest SINR when no interferences is present. The proposed FDL robust beamformer works better for higher number of interferences.

## 5. CONCLUSION:

In this paper, the relative comparison in radiation pattern of conventional delay and sum beamformer and optimal beamformer under the absence and presence of mutual coupling have been addressed. We also investigated the performance degradation due the presence of MC of the optimal beamformer such as SINR, main beam shape, nulls level, sidelobe level, etc. A robust technique using FDL method has been developed by using the noise power. We have shown that the proposed algorithm add the fixed amounts of diagonal loading factor. The excellent performance of ourproposed algorithm has been explained by a number of matlab simulations and which will be very helpful for real-time implementation. The interference cancellation by the proposed algorithm can be used to cancel Jammer in wireless communication.

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