Gaussian Noise Filtering From ECG Signal Using Improved Kalman Filter


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Abstract: An electro cardogram (ECG) signal is a series of waves and deflections which represents the electrical activity of the heart over time. ECG signal can be contaminated by different forms of noises. These noises mislead the ECG annotators from accurate identification of the ECG signal features. Conventional filters are used to reduce the noise components in ECG signal from different unwanted frequency components. But this filtering process is not efficiently work on the contaminated ECG signals. In biomedical signals reduction of noise is difficult with the fixed coefficients filters, because these signals are random in nature and not exact known depending on the time. To overcome this type of problem we require an adaptive algorithm technique. In this paper we present improved Kalman filter developed with a state space model and autocorrelation least squares(ALS) technique to estimate the state variables, including the Gaussian noise approximation from the previous values of the original ECG signal. This filter enhances the quality of the ECG signal and shows the good convergence properties. The results have been concluded with the MIT-BIH arrhythmia data base and MATLAB software.

Keywords: ECG signal, Gaussian noise, Adaptive algorithm, Kalman filter, SNR.

I. INTRODUCTION

Electrocardiogram (ECG) signal is an electrical manifestation of the contractility of the heart. Ambulatory ECG signal recordings obtained by placing electrodes on the body chest using invasive method. The ECG signal frequency ranges from 0.5 to 100 Hz [1]. ECG analysis is very important for the evolution of cardiac arrhythmia and it guides the condition of a normal heart beat. Fig1 represents an example of a normal ECG signal.

ECG is includes the valuable information about heart condition, but it is frequently corrupted by various noises such as power line interferences and harmonics from power mains, muscle contraction, respiration and electromyogram is mixed with ECG, baseline wander noise is occurred due to the variable connection between the skin and the connected electrodes [2]. Noise affects at both low-frequency and high frequency components. This noise component reduces the precision and accuracy of an ECG signal. Hence noise reduction from ECG signals is very important research and studied widely from many years.

Fig 1: A typical ECG signal
II. LITERATURE SURVEY

ECG noise cancellation has long puzzled the research community. Normal noise reduction methods are based on the standard filtering process techniques [1]-[4]. The ANC based on the Normalized LMS (NLMS) algorithm has a higher convergence rate than the algorithm used in normalized LMS algorithm [8]. The NLMS algorithm is also based on a fixed step size as used in the LMS algorithm. The authors used the Recursive Least Square algorithm (RLS) in the ANC to eliminate the ECG noise [8]. The main disadvantage of the RLS algorithm is its very high computational complexity, while it provides fast convergence rate. Even though there were number of advanced signal processing techniques are applied to the study of the noisy ECG signals, such as adaptive filter, wavelet transforms, and independent component analysis [7]. Some of the artifact problems and low signal to noise ratio problems are arise during the filtering of ECG signals may be reduced with the simple, frequency selective filtering techniques. Still it is very important and interesting approach to study about the noisy ECG filtering characteristics.

III. KALMAN FILTER DERIVATION

The Kalman filter is a recursive predictive filter that is based on the use of the recursive algorithms and the state space model techniques. The Kalman filter is developed with a set of mathematical equations that implements a predictor-corrector type estimator. It is an optimal data processing algorithm in the sense that it minimizes the estimated error covariance when some presumed conditions are met [9].

3.1 The Process to Be Estimated:

The Kalman filter is used to solve the problem of estimating the state ‘x’ of a controlled process that is regulated with the linear stochastic difference equation,

\[ x_k = A x_{k-1} + Bu_k + w_{k-1} \]  

And with a measurement ‘y_k’, it is

\[ y_k = Hx_k + v_k \]

The random variables in above equations \( w_k \) and \( v_k \) represents the process and measurement noise respectively. These noises are assumed to be white process with normal probability distributions and independent of each other [10],

\[ p(w) \sim N(0,Q) \quad \text{and} \quad p(v) \sim N(0,R) \]

The measurement noise covariance \( R \) and process noise covariance \( Q \) matrices may be change with each time step, however here we assume that they are constant.

3.2 The Computational Origins of the Kalman Filter:

We define \( x_k^- \) as our a priori state estimation at the step k given the information of the process prior to step k, and assume \( x_k^+ \) our a posteriori state estimate at the step k with given measurement \( y_k \). Then we can define a priori and a posteriori estimate errors as

\[ e_k \equiv x_k - x_k^- \]
\[ e_k^+ \equiv x_k - x_k^+ \]

a priori estimate error covariance is given by

\[ P^-_k = E[e_k^- e_k^-^T] \]

and a posteriori estimate error covariance is given by

\[ P_k = E[e_k e_k^T] \]

A posteriori state estimate is given by

\[ x_k^+ = x_k^- + K(y_k - Hx_k^-) \]

The difference in above equation is referred as the residual or measurement innovation. The residual reflect the divergence between the actual measurement and the predicted measurement. The \( nxm \) matrix \( K \) is chosen to be as the
blending factor or gain that reduces the a posteriori error covariance equation [9]. The resulting gain \( K_k \) that minimizes equation (3.7) is given

\[
K_k = P_k^- H^T (HP_k^- H^T + R)^{-1}
\]  

(3.8)

### 3.3 Kalman Filter Algorithm:

The Kalman filter is estimate the process state at a specific time and then it obtains feedback in term of measurement. The Kalman filter equations are categorized into two groups: time update equations and measurement update equations. Time update equations are mainly used for projecting forward in time domain of the current state and error covariance estimate to obtain a priori estimate for the next time step.

Measurement update equations are used for the feedback that means for integrating a new measurement into a priori estimate to obtain an improved a posteriori estimate. Table 3.1 and 3.2 shows specific equations for the time update and measurement update equations.

**Table 3.1: Kalman filter time updating equations**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3.9)</td>
<td>Time Update (Predict)</td>
</tr>
<tr>
<td>(3.10)</td>
<td>Measurement Update (Correct)</td>
</tr>
<tr>
<td>[ x_k^- = A\hat{x}<em>{k-1} + Bu</em>{k-1} ]</td>
<td>1. The state ahead projection</td>
</tr>
<tr>
<td>[ P_k^- = AP_k^- A^T + Q_k ]</td>
<td>2. The error covariance ahead projection</td>
</tr>
</tbody>
</table>

**Table 3.2: Kalman filter time update equations**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3.11)</td>
<td>Time Update (Predict)</td>
</tr>
<tr>
<td>(3.12)</td>
<td>Measurement Update (Correct)</td>
</tr>
<tr>
<td>[ K_k = P_k^- H^T (HP_k^- H^T + R_k)^{-1} ]</td>
<td>1. Kalman gain computation</td>
</tr>
<tr>
<td>[ x_{k+1}^- = x_k^- + K_k (y_k - H\hat{x}_k^-) ]</td>
<td>2. Updating the estimate with Measurement ( z_k )</td>
</tr>
<tr>
<td>[ P_{k+1} = (1 - K_k H_k) P_k^- ]</td>
<td>3. Updating of the error covariance</td>
</tr>
</tbody>
</table>

![Fig 2: A complete structure of Kalman filters operation.](image)

Initial estimation at \( k=0 \) and output values at \( k \) will be the inputs for \( k+1 \).
it makes a practical implementation much more executable than an implementation of a Wiener filter which is used to operate on each estimate for all of the arriving data directly. The Kalman filter uses recursive conditions and the current state estimation on all previous measurements. Figure 4.2 shows a complete representation of the operation of the filter which is combining with the high-level picture of Fig 4.1 with the state equations from table 4.1 and table 4.2.

IV. PROPOSED ALGORITHM

4.1 Improved Kalman Filter Approach:

The derived Kalman filter implementation is often difficult due to the problem of obtaining a good estimate of the process and measurement noise covariance matrices \(Q_k\) and \(R_k\). More research work has been done in this field of getting good estimates of this covariance from data. One of the important and practical approach to get good estimates is the “Auto covariance least squares (ALS)” technique that uses the time lagged auto covariance of routine operating data to estimate the covariance. The error noise covariance noise is calculated by using the ALS is as follows

\[
P_{k|k} = \text{cov}(x_k - x_{k|k})
\]

(3.11)

After calculating and collecting the error vectors we get,

\[
P_{k|k} = \text{cov}((I - K_k H_k)(x_k - x_{k|k-1}) - K_k v_k)
\]

(3.12)

Since the measurement error vector \(v_k\) is uncorrelated with the remaining terms, it becomes

\[
P_{k|k} = (I - K_k H_k)P_{k|k-1}(I - K_k H_k)^T + K_k R_k K_k^T
\]

(3.13)

This formula sometimes known as the Joseph form of the covariance update equation and it is valid for any value of \(K_k\). The eq (3.13) is very important if a non-optimal Kalman gain is advisedly used.

\[
K_k = P_{k|k} H_k^T (HP_{k|k} H_k^T + R_k)^{-1}
\]

(3.14)

It is straight forward equation to prove that the resultant Kalman gain is increased. Hence, more emphasis is put on newly arriving data. Due to increase in kalman gain the Gaussian noise reduces efficiently and the signal to noise ratio is increases in proposed algorithm compare to the Kalman filter approach.

V. RESULTS

The following figures represent the simulation results for different contaminated noisy ECG signals which are applied to the Kalman filter and Improved Kalman filter.

In Fig.1 ‘clean ECG signal’ shows original ECG signal which is obtained from the MIT-BIH arrhythmia data base. Gaussian white noise is used as the noise source and embedded in the ECG signal. In this study, the Gaussian noise signal is generated by Matlab code awgn.m and the contaminated ECG signal is represented by ‘Noisy ECG signal’ with 18 dB input SNR. The next two signals are output of Kalman filter and IKF approach.

Fig 1: Simulation results of Kalman filter and Improved KF for contaminated noisy ECG signal with input SNR 18dB
In Fig. 2 the considered noisy ECG signal is with input SNR is 22dB and corresponding outputs are shown.

**Fig 2: Simulation results of Kalman filter and Improved KF for contaminated noisy ECG signal with input SNR 22dB**

In Fig. 3 the determined input SNR is 25dB and corresponding outputs for both of the filters are shown.

**Fig 3: Simulation results of Kalman filter and Improved KF for contaminated noisy ECG signal with input SNR 25dB**

In Fig. 4 the acquired input signal is the sinus arrest signal. This signal occurs when the SA node stops firing, causes a pause in electrical activity. The seriousness of sinus arrest depends on the length of the pause. The patient will require immediate treatment.

**Fig 4: Simulation results of Kalman filter and IKF for noisy sinus arrest ECG signal with input SNR 18dB**
In Fig.5 the obtained input signal is the wandering atrial pacemaker (WAP). This signal is a rhythm in which the pacemaker site shifts between the SA node, atria or the AV junction. The P wave configuration changes in appearance during the pacemaker shift. It related to some types of organic heart disease and drug toxicity.

Fig 5: Simulation results of Kalman filter and Improved KF for noisy WAP ECG signal with input SNR 18dB

The summarizing of signal parameters for different contaminated noisy ECG signals for first three figures are shown in table I.

Mean square error (MSE) between filter ECG output and clean ECG was used to measure the filter performance. The lowest MSE value represents better filtering performance. Improved KF algorithm achieves a minimum MSE. The higher SNR values showed less noise part embedded and a cleaner ECG signal.

Table 1: Signal parameters for various contaminated ECG signals with different input SNR

<table>
<thead>
<tr>
<th>18 db ECG signal</th>
<th>22 db ECG signal</th>
<th>25 db ECG signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kalman Filter</td>
<td>Improved KF</td>
<td>Kalman Filter</td>
</tr>
<tr>
<td>MSE</td>
<td>0.0046</td>
<td>0.0019</td>
</tr>
<tr>
<td>SNR</td>
<td>23.34</td>
<td>29.54</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.3343</td>
<td>-0.3337</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0331</td>
<td>0.0308</td>
</tr>
<tr>
<td>STD</td>
<td>0.1820</td>
<td>0.1754</td>
</tr>
</tbody>
</table>

The following table shows the summarizing of signal parameters for abnormal ECG signal such as sinus arrest and wandering atrial pacemaker (WAP).

Table 2: Signal parameters for contaminated ECG signals with 18dB input SNR

<table>
<thead>
<tr>
<th>Noisy ECG signal</th>
<th>Sinus arrest ECG signal</th>
<th>WAP ECG signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kalman Filter</td>
<td>Improved KF</td>
<td>Kalman Filter</td>
</tr>
<tr>
<td>MSE</td>
<td>0.0046</td>
<td>0.0049</td>
</tr>
<tr>
<td>SNR</td>
<td>23.34</td>
<td>24.11</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.3343</td>
<td>-0.3660</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0331</td>
<td>0.0322</td>
</tr>
<tr>
<td>STD</td>
<td>0.1820</td>
<td>0.1794</td>
</tr>
</tbody>
</table>

The following graphs show the performance for the both of the kalman filter and improved KF algorithms using different contaminated ECG signals. It shows the better output SNR values. The statistical values are considered from the Table I.
Fig 6: Mean square error values for different contaminated ECG signals

Fig 7: SNR values for different contaminated ECG signals

Fig 8: Mean values for different contaminated ECG signals
VI. CONCLUSION

In this theory, the improved Kalman filter has been developed and simulated with the estimation of adaptive noise covariance on different contaminated ECG signals. Also evaluated to assess whether the IK filter has ability to enhancing the SNR value of the signals. At the same time this filter preserves the morphological variations occurred during the recording of the ECG signal. This filter operates with the estimation of the process and measurement noise covariance using ALS technique. This filter has ability to adapt its estimated noise covariance quickly to correspond the output of the filter to the next input. The performance evaluation of this filter is better than a similar derived Kalman filter with the fixed values of process noise covariance.

REFERENCES


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