# An Innovative Decaying DC Component Estimation Algorithm for Digital Relaying

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Abstract—We propose a new decaying dc component estimation algorithm for digital relaying. Fault currents tend to include a dc decaying component. This component decreases the accuracy and speed of the protection relay operation. The proposed algorithm can estimate and eliminate the dc decaying component from fault current signals after one cycle from the fault instant. Also, it can be applied to a conventional discrete Fourier transform to calculate phasor quantities of fault currents in a digital protection relay. In the proposed algorithm, the dc decaying magnitude and time constant are estimated exactly by integrating fault currents during one cycle. The dc decaying component is eliminated by subtracting the dc value at each sampling instant. To verify the performance of the proposed algorithm, we performed a dc component estimation test and distance protection test using PSCAD/EMTDC. The results of the PSCAD/EMTDC simulation showed that the proposed algorithm can estimate dc components exactly from fault currents and can be applied to digital protection relays for phasor extraction.

*Index Terms*—DC estimation, dc magnitude, dc time constant, digital protection, discrete Fourier transform (DFT), fault current.

# I. INTRODUCTION

HEN a fault occurs on power systems, not only are the fundamental components included in fault currents, but so are the decaying dc and harmonic components. But the protection system only uses the fundamental component for fault discrimination. Other components usually are obstacles in extracting the fundamental component from the current's waveform. In digital protection relaying, the discrete Fourier transform (DFT) is the most preferable method to extract the fundamental phasor quantities from waveforms [1]. DFT has immunity from harmonic components and has a relatively fast response time for the fundamental component calculation. However, the DFT is not immune from the dc component, and the decaying dc component in the fault current can cause undesirable oscillations in the DFT results [2], [3]. Since these undesirable oscillations can cause abnormal operation of the protection system, especially distance protection, practical digital relaying schemes require additional techniques to reduce the dc component effects in the DFT results.

Up till now, many research studies were conducted to remove the dc component from fault current waveforms for protection relaying [2]-[14]. These research efforts can be categorized into two methods. One is the dc component filtering method, which extracts the fundamental components only from the original signal without a dc component calculation. In [2], a mimic filter was proposed to remove the decaying dc component over a wide range of time constants. Recently, an adaptive compensation method to remove a decaying dc offset component from the fault signals has been described in [4]. The dc component filtering method achieves satisfactory performance when the time constant of the dc component is equal to the time constant of the filter. However, this is usually not the case in power systems because the time constant and magnitude of the decaying dc component are characterized by the system configuration, fault resistance, and fault position. The other method is the dc estimation method, which calculates the dc component and subtracts it from the original signal to obtain the fundamental component only. A modified DFT algorithm to efficiently compute and eliminate the dc component using full-cycle or half-cycle data windows has been proposed in [5]. The technique for removal of a decaying dc offset on phasor estimates using the DFT is described in [6]. An adaptive phasor estimation algorithm to suppress the effect of an exponential decaying dc component based on the weighting least error square (LES) technique is proposed [7]. This approach has the advantage in that it removes the decaying dc offset regardless of its initial magnitude and time constant, while the disadvantage of the method is that the dc component is determined by the complex calculation procedure. The dc estimation method has difficulty in calculating the exact dc component from the original signal and requires more calculation times. But, if the dc component can be estimated, this method is not affected by the power system configuration, fault resistance, and fault location. The estimated dc component can be used as additional information for fault discrimination. In this method, the accuracy of the dc component estimation and required calculation time, which can be applied in practical relaying schemes, are very important.

In this paper, we describe a new and unique algorithm to estimate and eliminate the decaying dc component in a fault current signal. We also describe the mathematical derivation of the proposed algorithm. The magnitude and time constant of the dc component were estimated by integrating the fault current. The dc-removed current signal was obtained by eliminating the dc component from the fault current at each sampling instant. We evaluated the performance of the proposed algorithm in the time domain using PSCAD/EMTDC. For evaluation of the algorithm, we performed dc component estimation tests with several

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dc component cases. The distance protection test was performed with a sample power system. The results of the test cases showed that the proposed algorithm can estimate the dc component exactly from fault currents and can be applied to digital protection schemes for phasor extraction.

This paper is organized as follows. Section II describes the mathematical derivation of the proposed algorithm to calculation the exact dc component from the original signal. Section III further provides details regarding the implementation needed for the dc component estimation. Numerical results of the static and dynamic simulation tests for the various events are described in Section IV. We discuss some comments about future work and conclusions in Section V.

### II. PROPOSED DC COMPONENT ESTIMATION ALGORITHM

Generally, the fault current not only has a fundamental component but also harmonics and a decaying dc component [6]–[8]. Fundamental and harmonic components can be represented as a sinusoidal function. The decaying dc component can be represented as a decaying exponential function. So the fault current can be mathematically expressed

$$i(t) = I_0 e^{-t/\tau} + \sum_{k=1}^p I_k \sin(k\omega_1 t + \theta_k)$$
(1)

where  $I_0$  is the magnitude of the decaying dc offset,  $\tau$  is the time constant of the decaying dc offset, k is the harmonic order,  $I_k$  is the magnitude of the kth harmonic component,  $\theta_k$  is the phase angle of the kth harmonic component, and P is the maximum harmonic order.

If (1) integrates during one period (T), the integral of the second term in (1) is zero, and only the integral of the first term, which is related to the decaying dc component, remains

$$\int_{t-T}^{t} i(t) dt = \int_{t-T}^{t} \left[ I_0 e^{-t/\tau} + \sum_{k=1}^{p} I_k \sin(k\omega_1 t + \theta_k) \right] dt$$
$$= \int_{t-T}^{t} I_0 e^{-t/\tau} dt = \left[ -I_0 \tau \cdot e^{-t/\tau} \right]_{t-T}^{t}$$
$$= -I_0 \tau \cdot e^{-t/\tau} (1 - e^{T/\tau})$$
$$= Z(t).$$
(2)

In (2), let the integral of the dc component during one period at time t be Z(t). Then,  $Z(t + \Delta t)$ , which is represented as the integral of the dc component after a small time step, is expressed

$$Z(t + \Delta t) = -I_0 \tau \cdot e^{-(t + \Delta t)/\tau} (1 - e^{T/\tau})$$
  
=  $-I_0 \tau \cdot e^{-t/\tau} (1 - e^{T/\tau}) \cdot e^{-\Delta t/\tau}$   
=  $Z(t) \cdot e^{-\Delta t/\tau}$ . (3)

From (3), if we know the integrals of fault current during one period, the following equations can be used to calculate the time constant ( $\tau$ ) and magnitude ( $I_0$ ) of the decaying dc component:

$$\tau = -\frac{\Delta t}{\ln \frac{Z(t+\Delta t)}{Z(t)}} \tag{4}$$

$$I_0 = \frac{Z(t)}{-\tau \cdot e^{-t/\tau} (1 - e^{T/\tau})}.$$
 (5)

In (4) and (5), to obtain Z(t) and  $Z(t + \Delta t)$ , we should only calculate the integral of the measured fault current at t and  $t + \Delta t$ .

### III. IMPLEMENTATION OF THE ALGORITHM

In a practical digital relaying scheme, all of the calculations are performed in a discrete time base using sampled data and should be completed in each sampling period. For the practical application of the algorithm, fast calculation time is required. In (4), it is clear that the decaying dc component can be mathematically calculated. However, since the calculation of the natural logarithm should be performed for every sample, it can be a computational burden from the practical perspective. In order to reduce the computational load, the time constant can be obtained by using a Taylor series expansion in (4)

$$e^{-\Delta t/\tau} = 1 + (-\Delta t)\frac{1}{\tau} + (-\Delta t)^2 \frac{1}{\tau^2} + \dots = \frac{Z(t + \Delta t)}{Z(t)}.$$
 (6)

In (6),  $\Delta t$  is the sampling period, so  $\Delta t$  is much lower than the time constant of the power system. Thus, we can use only the first two terms to calculate the time constant and simplify the time constant to be

$$e^{-\Delta t/\tau} = 1 + (-\Delta t)\frac{1}{\tau} = \frac{Z(t+\Delta t)}{Z(t)}$$
 (7)

$$\frac{1}{\tau} = \left(1 - \frac{Z(t + \Delta t)}{Z(t)}\right) \cdot \frac{1}{\Delta t}.$$
(8)

Also, the magnitude of the decaying dc component can be calculated by applying t = T to (5). Therefore, we acquire the following equation:

$$I_0 = \frac{Z(T)}{-\tau (e^{-T/\tau} - 1)}.$$
(9)

But the calculation of the magnitude of the dc component is not necessary to remove the dc component from the fault current signal. If we know the time constant, we can calculate the dc value directly at a sampling instant. From (5), the dc value at time t is calculated with a time constant and integral of the fault current. The dc value for the next sampling instant can be calculated by multiplying previous dc values by an exponential increment as shown in

$$I_{0}e^{-t/\tau} = \frac{Z(t)}{\tau(e^{T/\tau} - 1)}$$
(10)  
$$I_{0}e^{-(t+\Delta t)/\tau} = \frac{Z(t+\Delta t)}{\tau(e^{T/\tau} - 1)} = I_{0}e^{-t/\tau} \cdot e^{-\Delta t/\tau}$$
$$= \frac{Z(t)}{\tau(e^{T/\tau} - 1)} \cdot e^{-\Delta t/\tau}.$$
(11)

Since the proposed algorithm can estimate a dc value after one cycle, by subtracting the calculated dc value from each of the sampled data in buffers which contain one cycle of sample data, and applying these results to the DFT, we can extract the fundamental component without any dc components. Overall, the calculation procedure to extract the fundamental component of sampled data is shown in Fig. 1.



Fig. 1. Procedure for the fundamental component calculation.

## **IV. TEST RESULTS**

In order to verify the performance of the proposed algorithm, two types of simulation tests were performed with PSCAD/EMTDC [15]. The first simulation test was a static test. In this test, several sampled signals, which contained a dc component, were applied to verify the performance of the dc component. The calculated time constants and fundamental components were compared to the applied signals. The second simulation test was a dynamic test. In this test, the proposed algorithm was applied to the distance relaying scheme in the sample power system and the performance of the distance relaying was compared to the cases of a conventional DFT and cosine-based method [12], [16], [17].

## A. Static Simulation Test

A static simulation test was performed to evaluate the performance of the dc magnitude and time constant calculation algorithm. Test signals consisted of a fundamental component and a dc component with different magnitudes and time constants. The ratio of the magnitude of the fundamental component and the decaying dc component was set to 0.2, 0.4, 0.6, 0.8, and 1.0 p.u. The time constants used for the performance evaluation were 5, 25, 50, 100, 150, and 200 ms. Also, the sampling rate was set to 64 samples per cycle.

Table I shows the estimated time constants using the proposed algorithm for the applied time constant and ratio of magnitude changes. It can be seen from Table I that the estimated time constants of the test signal exhibited good agreement with the applied value. In the case of  $\tau = 5$  ms and ratio = 0.2, the error between the applied and estimated time constant was 3.14%. Also, Table II shows the estimated time constant and frequency changes to investigate the offnominal frequency system operation in the power system,

Fig. 2 shows the applied signal and estimated dc value in the time-domain simulation. It took one cycle to estimate the dc value from the first appearance of the dc decaying component.

Fig. 3(a) and (b) shows the time-domain responses of the fundamental value of the applied signal using the proposed algorithm, conventional DFT, and the cosine-based method. In this

TABLE I Estimated Time Constants

$I_f/I_{dc}^*$ -	The time constant of input signal (ms)						
	5	25	50	100	150	200	
0.2	5.157	25.173	50.195	100.225	150.235	200.223	
0.4	5.143	25.148	50.155	100.162	150.158	200.144	
0.6	5.138	25.139	50.141	100.140	150.133	200.118	
0.8	5.136	25.135	50.134	100.130	150.120	200.105	
1.0	5.135	25.132	50.130	100.123	150.112	200.097	

\* The ratio of the magnitude of the fundamental component and the decaying DC component.

 TABLE II

 ESTIMATED TIME CONSTANTS FOR OFFNOMINAL FREQUENCY

The frequency of input signal <sup>*</sup> (Hz)						
59.8	59.9	60.0	60.1	60.2		
24.844	25.024	25.139	25.171	25.188		
29.454	49.908	50.141	50.148	49.302		
	The frequ           59.8           24.844           29.454	The frequency of input           59.8         59.9           24.844         25.024           29.454         49.908	The frequency of input signal* (Hz)           59.8         59.9         60.0           24.844         25.024         25.139           29.454         49.908         50.141	The frequency of input signal* (Hz)         59.8       59.9       60.0       60.1         24.844       25.024       25.139       25.171         29.454       49.908       50.141       50.148		

<sup>\*</sup> I<sub>f</sub>/I<sub>dc</sub> = 0.6 pu



Fig. 2. Applied signal and calculated dc value ( $\tau = 25$  ms, ratio = 0.4).

case, the time constants of 25 ms and 150 ms with a 0.6 magnitude ratio were applied. In Fig. 3, the conventional DFT had an oscillation in the fundamental component and required more



Fig. 3. Time-domain responses of the proposed algorithm and conventional DFT for different time constants.

times to obtain a stable output. But the proposed algorithm extracted the fundamental component without any undesired oscillation in one cycle and delay to obtain a stable output.

From the results, the magnitude and time constant of a decaying dc component can be accurately calculated from the input signal by using simple mathematical expressions. Also, the performance of the proposed algorithm can meet the requirement of extensive relay studies for various systems and fault conditions.

# B. Dynamic Simulation Test

In order to demonstrate the effectiveness of the proposed algorithm used for the distance relay study, an extensive simulation on PSCAD/EMTDC was performed. As mentioned before, various conditions, such as fault locations and fault resistance, were considered in the test studies.

A set of simulation tests was verified using the configuration of the power system shown in Fig. 4. The simulated system was



Fig. 4. One-line diagram of a test system.

a 230-kV, 100-km transmission line with sources at both terminals. The model of the distance relay using the proposed algorithm was embedded into the model of the power system for the relay at bus 1. Sample numbers were set to 64 per period and the impedance characteristic was an *mho* type. The Zone 1 setting was to about 85%.

To examine the robustness of the proposed algorithm, four scenarios were defined and are shown in Table III. All scenarios assumed that the fault was initiated at 0.2 s and cleared at 0.3 s. Fig. 5(a) and (b) shows the time-domain response of the current signal using the proposed algorithm at the relay when a three-phase fault occurred at the middle of a transmission line for a fault resistance of 0.001 and 10  $\Omega$ .

Fig. 5(a) and (c) illustrates the time-domain response of the current signal for various fault locations of 50 and 80 km. Fig. 5(c) and (d) depicts the comparative results of the current estimation using two fault types, such as three-phase faults and single-line-to-ground faults.

In order to obtain a clearer picture of the performance of the two techniques, the apparent impedance as seen by the relay located at bus 1 for scenario A is shown in Fig. 6, when a phase-A-to-ground fault occurred at a fault distance of 80%. In contrast, Fig. 7 depicts the relay response for an external fault, phase-A-to-ground fault. The fault was set between bus 1 and the generator terminal. As shown in these figures, a distance relay adopting the proposed algorithm had no problems with the operations.

We observe these figures as follows.

- As shown in Fig. 5(a), the percentage deviations of the proposed algorithm and the conventional DFT and the cosine-based method from the final value were 0.4%, 14.5%, and 2.1% at t = 0.22 s, respectively. The conventional DFT exhibited overshoots in their outputs. We note that the proposed algorithm had a better accuracy response compared to the conventional DFT and cosine-based method.
- It can be seen as shown in Fig. 5(b) that the distance relay designed by the proposed algorithm took close to 0.015 s to converge to its final value, while the conventional DFT and the cosine-based method performances converged to the final value within 0.045 s and 0.056 s, respectively. From this comparison of time-domain responses for four scenarios, the proposed algorithm was faster than the conventional DFT- and cosine-based method.
- Through extensive fault simulations under various fault conditions, the proposed algorithm was effective in terms of the digital distance relay design.
- All of the aforementioned results clearly indicated that the proposed algorithm can accurately estimate a decaying dc component and quickly extract the fundamental frequency



Fig. 5. Time responses of the current signal of the different algorithms for various conditions.

# TABLE III Description of the Scenarios

Scenario	Fault conditions					
Scenario	Туре	Location (km)	Resistance (Ω)			
A	A-B-C-G	50	0.001			
В	A-B-C-G	50	10			
С	A-B-C-G	80	0.001			
D	A-G	80	0.001			

signal. Also, the proposed algorithm had better convergence and accuracy compared to the conventional DFT and cosine-based method.

# V. CONCLUSION

A new algorithm based on decaying dc component estimation and elimination for digital relaying has been presented. The proposed algorithm can obtain the dc magnitude and the time constant calculation which was estimated by integrating fault currents during one cycle. Thus, it can be used for extraction of the fundamental frequency component.



Fig. 6. Trajectories of the apparent impedance for scenario D.

A comprehensive set of simulation results through static and dynamic tests has shown that the proposed algorithm has faster convergence and better accuracy than the conventional DFT and



Fig. 7. Trajectories of the apparent impedance for the external fault.

cosine-based method. Furthermore, a distance relay employing the proposed algorithm provides a correct relay response. The proposed algorithm can be easily implemented and applied to fast digital distance relaying for transmission lines.

The task of expanding the dc component estimation during half a cycle is underway. Also, further work for validating, through comprehensive dynamic testing with a hardware platform adopting the proposed algorithm, is ongoing.

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