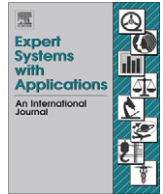




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# A deterministic annular crossover genetic algorithm optimisation for the unit commitment problem

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## ABSTRACT

One of the disadvantages of traditional genetic algorithms is premature convergence because the selection operator depends on the quality of the individual, with the result that the genetic information of the best individuals tends to dominate the characteristics of the population. Furthermore, when the representation of the chromosome is linear, the crossover is sensitive to the encoding or depends on the gene position. The ends of this type of chromosome have only a very low probability of changing by mutation. In this work a genetic algorithm is applied to the unit commitment problem using a *deterministic* selection operator, where all the individuals of the population are selected as parents according to an established strategy, and an *annular* crossover operator where the chromosome is in the shape of a ring. The results obtained show that, with the application of the proposed operators to the unit commitment problem, better convergences and solutions are obtained than with the application of traditional genetic operators.

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## 1. Introduction

In the commercial operation of an electricity market, the correct planning of generator units is of fundamental importance. The economic savings, together with efficiency in the use of energy resources, mean that new proposals to solve the unit commitment problem (UCP) continue to be sought (Padhy, 2004; Yamin, 2004).

The UCP has been solved using deterministic methods, such as Priority List (PL) (Senjyu, Miyagi, Saber, Urasaki, & Funabashi, 2006; Senjyu, Shimabukuro, Uezato, & Funabashi, 2003), Dynamic Programming (DP) (Ouyang & Shahidepour, 1991; Rong, Hakonen, & Lahdelma, 2009), Lagrangean Relaxation (LR) (Ongsakul & Pertchakaras, 2004; Zhai, Guan, & Cui, 2002) and Mixed Integer Linear Programming (MILP) (Carrión & Arroyo, 2006; Frangioni, Gentile, & Lacalandra, 2009; Zhai, Guan, & Yang, 2009). These methods are characterised principally by their speed and their capacity to handle large scale problems when the objective function is linear and when some constraints are not considered. Otherwise, with PL the quality of the final solution is not guaranteed; DP suffers the problem of dimensionality; with LR a feasible solution is not guaranteed; and with MILP it is difficult to achieve a balance between the efficiency and the accuracy of the model.

In the face of these disadvantages the search for new methods has focused on metaheuristic methods such as genetic algorithms

(GA) (Arroyo & Conejo, 2002; Damousis, Bakirtzis, & Dokopolous, 2004; Dang & Li, 2007; Dudek, 2004; Sun, Zhang, & Jiang, 2006; Swarup & Yamashiro, 2002), Simulated Annealing (SA) (Mantawy, Abdel-Magid, & Selim, 1998; Purushothama & Jenkins, 2003; Saber, Senjyu, Miyagi, Urasaki, & Funabashi, 2007; Simopolous, Kavatzia, & Vournas, 2006; Zhuang & Galiana, 1990) and Particle Swarm Optimisation (PSO) (Lee & Chen, 2007; Yuan, Nie, Su, Wang, & Yuan, 2009). Using these metaheuristic methods it is possible to find an optimal solution to complex problems which is their main advantage over deterministic methods. Due to their iterative nature however, metaheuristic methods require a large amount of computer time to find a solution near to the global optimum, especially in large-scale problems. Today many proposals are based on hybrid techniques (Aruldoss & Ebenezer, 2006; Padhy, 2001; Patra, Goswami, & Goswami, 2009; Yin-Wa, 2001) which exploit the advantages of both deterministic and metaheuristic techniques, making them attractive alternatives for solving the UCP.

GA is a global optimisation method which works well and efficiently on objective functions which are complex in terms of the nonlinearities and constraints imposed. One of the disadvantages of GA is premature convergence, because when the selection is based on the quality of the individual, the genetic information of the best individuals tends to dominate the genetic characteristics of the population. Another disadvantage results from the representation of chromosomes in string form, with the result that the genetic information at the ends of the chromosome tend to remain unaltered during crossover. In this chromosome, modification of the ends can only be achieved by the mutation operator.

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To avoid disadvantages of this type in UCP the use of a GA is proposed, which is based on a deterministic selection process and an annular crossover operator (Kuri, 1998). Thus a deterministic annular crossover genetic algorithm (DACGA) is proposed for the UCP. The chromosome of an individual is represented as a binary matrix of the operational states which the generator units may assume during the planning period. To carry out the deterministic selection process, the total cost of the programming period is used as the fitness of each individual. Then, the exchange of genetic information between two individuals with the annular crossover operator, a ring representation of the scheduling period of a generator unit from each individual is used. Finally, the proposed DACGA is applied to the UCP using test systems of 10, 38 and 45 generating units. The scheduling time horizon is chosen as one day with 24 intervals of one hour each.

## 2. Unit commitment formulation

The objective function and constraints associated with the unit commitment problem are the following.

### 2.1. Objective function

The mathematical model used as the objective function to obtain the unit commitment of thermal units is:

$$OF = \sum_{h=1}^H \sum_{n=1}^N (FC_n^h + SU_n^h + SD_n). \quad (1)$$

Where  $OF$  represents the total production cost for horizon planning,  $H$  is the total number of hours and  $N$  is the total number of units.

This objective function includes the fuel costs of unit  $n$  in hour  $h$  as a function of the power generated, normally represented by a quadratic equation as:

$$FC_n^h(P_n^h) = a_n + b_n P_n^h + c_n (P_n^h)^2. \quad (2)$$

The start-up cost is dependent on the number of hours during which the unit has been off ( $TOff_n$ ). Using the two-step function, the start-up cost function is given by:

$$SU_n^h = \begin{cases} HS_n, & \text{if } TOff_n \leq T_{Cold,n}, \\ CS_n, & \text{other wise.} \end{cases} \quad (3)$$

Where  $HS_n$  is the hot start cost,  $CS_n$  is the cold start cost

$$T_{Cold,n} = Tdn_n + CSH_n. \quad (4)$$

Where  $T_{Cold,n}$  is the number of hours that it takes for the boiler of unit  $n$  to cool down,  $Tdn_n$  is minimum downtime of unit  $n$  and  $CSH_n$  is the cold start hours

The shut-down cost values  $SD_n$  are generally considered to be constant.

### 2.2. Constraints

The optimisation of the objective function is subject to a number of system and unit constraints as follows.

- System power balance.

The total power generated by all  $On$  units must supply the load demand in the hour  $h$ .

$$\sum_{n=1}^N P_n = D^h. \quad (5)$$

- Demand and spinning reserve.

The maximum power generated by all  $On$  units must at least meet the demand plus the spinning reserve in hour  $h$ .

$$\sum_{n=1}^N PMax_n \geq D^h + R^h. \quad (6)$$

- Minimum load conditions.

The minimum power generated by all  $On$  units must be less than or equal to the demand in hour  $h$ .

$$\sum_{n=1}^N PMin_n \leq D^h. \quad (7)$$

- Minimum up and down times.

The total number of hours for which unit  $n$  has been running ( $TON_n$ ) must be greater than or equal to the minimum unit uptime ( $Tup_n$ ).

$$TON_n \geq Tup_n, \quad n \in N. \quad (8)$$

Similarly, the total number of hours for which unit  $n$  has been down ( $TOff_n$ ) must be greater than or equal to the minimum unit downtime ( $Tdn_n$ ).

$$TOff_n \geq Tdn_n, \quad n \in N. \quad (9)$$

- Generator technical limits

Each unit has a generation range which is represented as:

$$PMin_n \leq P_n \leq PMax_n, \quad n \in N. \quad (10)$$

- Unit initial status.

The initial status at the start of the scheduling period must be taken into account.

## 3. The proposed method

### 3.1. Overview of genetic algorithms

Genetic algorithms are robust search techniques inspired in genetics and in the processes of natural selection of individuals competing in the same environment. Those individuals best adapted to the environment tend to transmit their genetic information to future generations.

One of the advantages of genetic algorithms in optimisation problems is that they do not require any more information than that provided by the objective function of the problem. With this technique the process of searching for solutions is isolated from the characteristics of the objective function and the constraints associated with it.

The algorithm starts with the creation of a combination of coded structures called *Chromosomes* (solutions) which make up the initial population. The criterion which evaluates the quality of each *Chromosome*, is given by the *Fitness* corresponding to the evaluation of each individual for the objective function. Once the fitness of each of the individuals in the population is known, it is subjected to a *Selection* process in which the best evaluated individuals have a greater probability of being chosen as *Parents* for the exchange of genetic information called *Crossover*. Then a percentage of the *Offsprings* (individuals generated in the crossover) are subjected to the *Mutation* process in which a random change is generated in the chromosome. This mutation process provides greater diversity between the individuals in the population.

When the crossover and mutation processes are complete a new population is generated which replaces the original population. This must be repeated until one of the convergence criteria defined for the problem is met. Each of these cycles is known as a *Generation*.

### 3.2. Deterministic selection

Traditionally, selection is based on fitness of the individuals, and in this way the individuals with greater fitness have a higher probability of being chosen for the crossover. In deterministic selection, also known as *Vasconcelos* (Kuri, 2004), a strategy is imposed in which individuals with better fitness are crossed with those of worse fitness.

For a population of  $K$  individuals ordered in descending order of fitness, the deterministic selection operator is shown in Fig. 1, in which the pairs of parents which are subjected to the crossover operator will be  $(1, K)$ ,  $(2, K - 1)$ , down to the last pair formed by  $(K/2, K/2 + 1)$ .

### 3.3. Annular crossover

Once two individuals have been selected, they are subjected to the crossover process in which genetic information is exchanged.

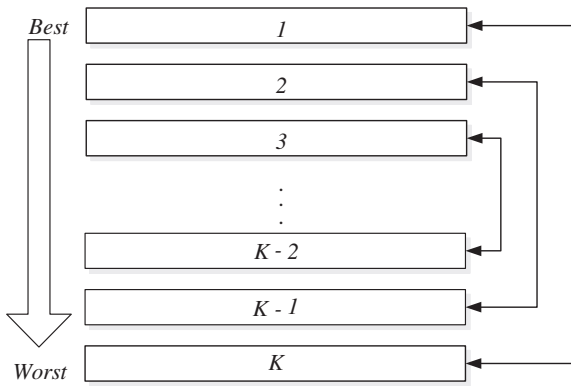


Fig. 1. Deterministic selection.

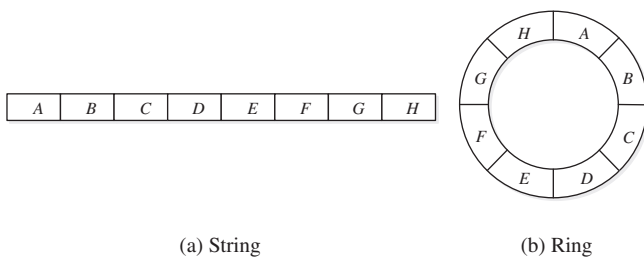


Fig. 2. Chromosome representation.

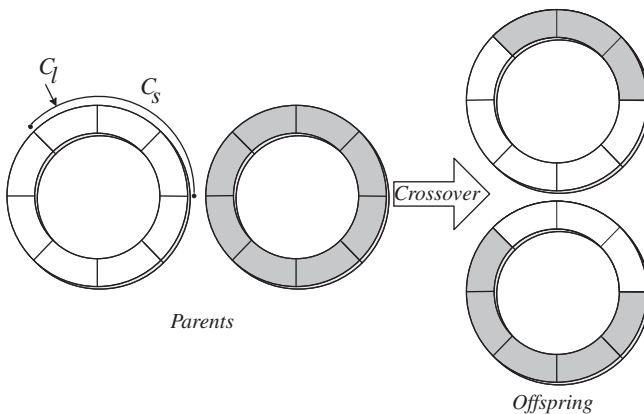


Fig. 3. Annular crossover.

Traditionally, in GA a linear crossover is performed on the chromosome represented as a string, as shown in Fig. 2a. However in an annular crossover the chromosome is represented as a ring, as shown in Fig. 2b.

The annular crossover is carried out by defining a number  $C_l$  which indicates the crossover locus. This number is in the range  $[1, L - 1]$ , where  $L$  is the length of the chromosome. Furthermore a number must be defined to establish the length of the semi-ring  $C_s$  which is exchanged during the crossover. The term semi-ring refers to a sector of the ring with a length in the range  $[1, L/2]$ . For the exchange of genetic information to be feasible, the length of the semi-ring must be the same in both chromosomes. The annular crossover operator described is shown in Fig. 3.

## 4. Deterministic annular crossover genetic algorithm optimisation (DACGA) for UCP

To resolve the UCP using the DACGA method proposed, the solution may be represented, as shown in Fig. 4, as a matrix of states of order  $N \times H$  where  $N$  is the total number of generating units and  $H$  is the total number of hours in the study period. A binary code is used in which 1 represents state of the unit as *On* and 0 represents the state of the unit as *Off*.

### 4.1. Deterministic selection for UCP

For UCP, the fitness of each individual corresponds to the total cost during the whole scheduling period, including the total fuel cost and the total start-up cost of the units. The total fuel cost is obtained with the lambda iteration method as an Economic Load Dispatch (ELD) sub-problem. The deterministic selection process can be represented as shown in Fig. 5.

### 4.2. Annular crossover for UCP

The crossover allows part of the genetic information to be exchanged between two possible solutions to the UCP. For the annular crossover operator to be applied, the programming period

		hour						
		1	2	3	4	...	23	24
unit	1	1	1	1	1	...	1	1
	2	0	1	1	1	...	0	0
	3	0	0	0	0	...	1	1
	⋮	⋮	⋮	⋮	⋮		⋮	⋮
		1	1	0	0	...	0	1
	N	0	0	0	0	...	0	0

Fig. 4. Solution representation.

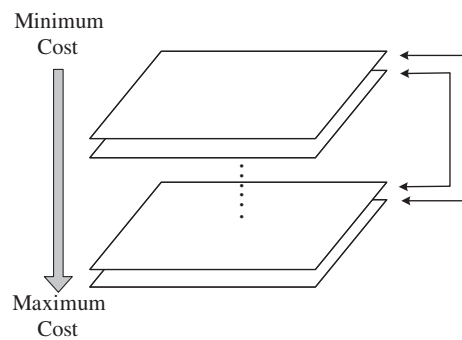


Fig. 5. Deterministic selection for UCP.

		hour							
		1	2	3	4	...	23	24	
unit	1	1	1	1	1	...	1	1	n
	2	0	1	1	1	...	0	0	
	3	0	0	0	0	...	1	1	
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
	N	1	1	0	0	...	0	1	
		0	0	0	0	...	0	0	

		hour							
		1	2	3	4	...	23	24	
unit	1	1	1	1	1	...	1	1	m
	2	0	1	1	1	...	0	0	
	3	1	0	0	0	...	1	0	
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
	N	0	0	0	0	...	0	0	
		0	0	0	0	...	1	1	

Fig. 6. Units for the crossover operator.

of a randomly chosen generator unit is represented as the ring chromosome. The annular crossover for the UCP is performed by the following steps.

**Step 1.** From each parent selected, a unit  $n$  and a unit  $m$  are chosen at random, uniformly distributed over  $[1, N]$ . Fig. 6 shows an example of the choice of units  $n$  and  $m$  from the parents involved in the crossover.

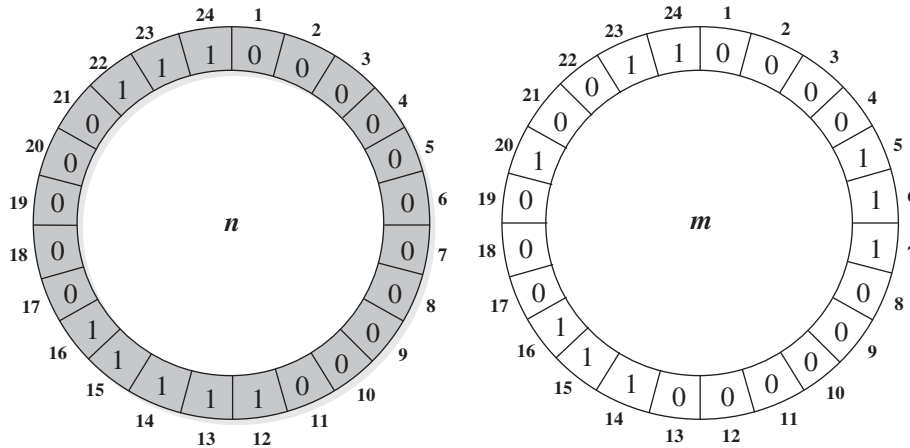
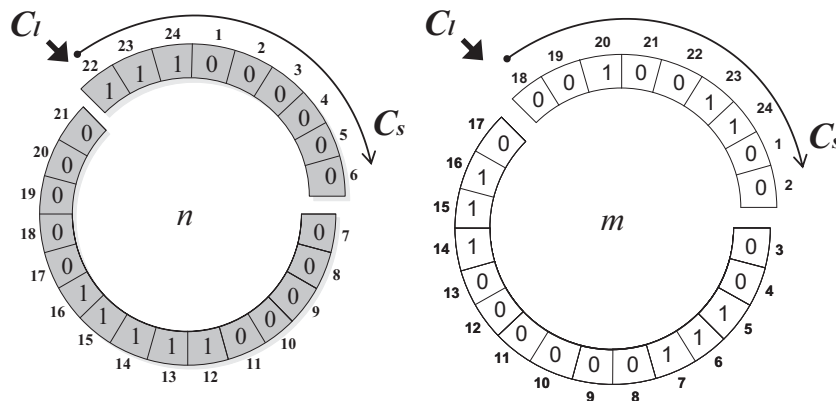
Fig. 7. Ring representation for units  $n$  and  $m$ .

Fig. 8. Semi-rings for the annular crossover.

**Step 2.** Define the scheduling of the chosen units by a ring representation as shown in Fig. 7.

**Step 3.** Generate the crossover point  $C_l$  and the length of the semi-ring  $C_s$  at random. In this case the length of the chromosomes is equivalent to the 24 h planning period. Fig. 8 shows an example of the resulting semi-rings, where  $C_l$  is 22 for unit  $n$  and 18 for unit  $m$ .  $C_s$  corresponds to a total of 9 h of planning to be exchanged.

**Step 4.** Exchange the genetic information in the semi-rings. Continuing with the example, Fig. 9 shows the new genetic information for units  $n$  and  $m$ .

**Step 5.** Return to the linear representation of the new scheduling of units  $n$  and  $m$  to incorporate this new genetic information into the chromosomes and generate the new offsprings.

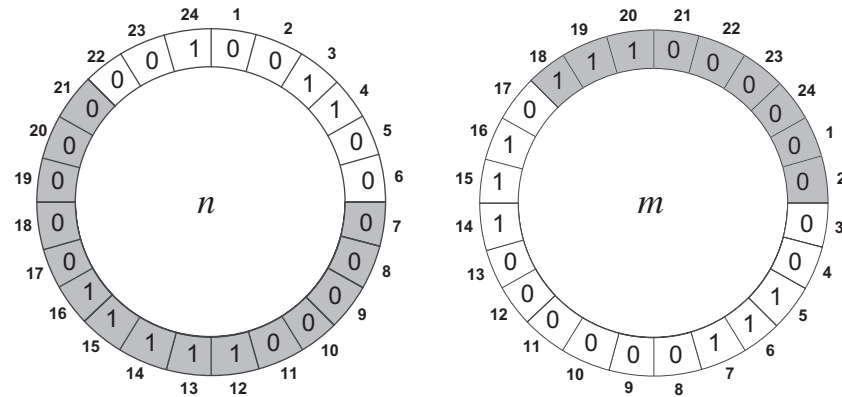
**Step 6.** End of crossover. Once the number of individuals in the population is complete, the annular crossover operator terminates.

#### 4.3. Mutation

A mutation probability  $P_m$  is defined to modify the genetic information in the chromosome. This modification of the genetic information changes just one randomly selected bit of the matrix chromosome from 1 to 0, or vice versa.

#### 4.4. Elitism

The object of maintaining a number of the best individuals in the next generation is so as not to lose the genetic information of

Fig. 9. New scheduling for units  $n$  and  $m$ .

the individuals with the best fitness, and thus increase the speed of convergence. In this way the proposed DACGA includes a certain degree of elitism, by which the best individuals of the population are kept to form part of the next generation.

#### 4.5. Repair mechanism

All the individuals of the new population are subjected to a mechanism intended to repair violations of the constraints of minimum start-up and shut-down times. This process is only carried out in one randomly selected generating unit.

### 5. Numerical examples

Test systems of 10, 38 and 45 generating units are used. Just as in the proposals in which these are used, a spinning reserve of 10% of demand is assumed in the systems of 10 and 45 units, while for the system with 38 units the spinning reserve assumed is 11% of demand.

When working with GA, the most common parameters, such as probability of mutation, probability of crossover, and population size, are established by sensitivity analysis. For the proposed DACGA the 10-unit system is used for sensitivity tests for the parameters of probability of mutation, elitism and population size. Moreover, this analysis is intended to find the parameters which will produce the best results in terms of convergence on the best solution. Figs. 10–12 show the results of 20 trials done for each value of the parameters under study.

Fig. 10 shows the sensitivity in the total cost with respect to the probability of mutation when the population size is 50 individuals, and full elitism is used.

The sensitivity in the total cost with respect to elitism is shown in Fig. 11. In this case, the population size is 50 individuals and the probability of mutation is 0.01.

Finally, Fig. 12 shows the sensitivity in the total cost with respect to the population size when the probability of mutation is 0.01, and full elitism is used.

From the sensitivity analysis no clear dependence is observed in the results obtained with respect to the probability of mutation. For elitism it is observed that the results improve when the number of the best individuals maintained in the next generation is increased. If a strategy of full elitism is adopted, it does not give rise to the problem of the best individuals tending to dominate the genetic characteristics of the population due to the use of deterministic selection. Finally, with respect to population size, it is observed that with less than 50 individuals the best results are not obtained. The parameters finally used in the DACGA are:

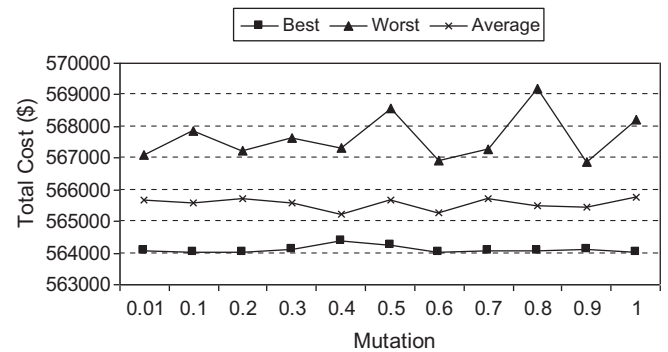


Fig. 10. Sensitivity respect to mutation.

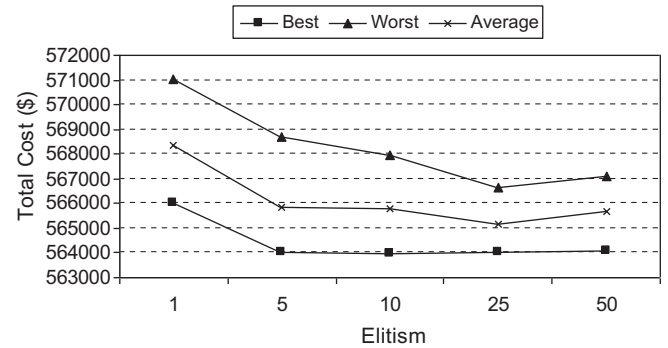


Fig. 11. Sensitivity respect to elitism.

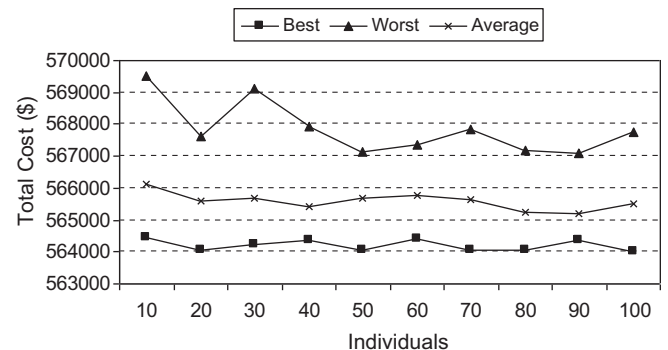


Fig. 12. Sensitivity respect to population size.

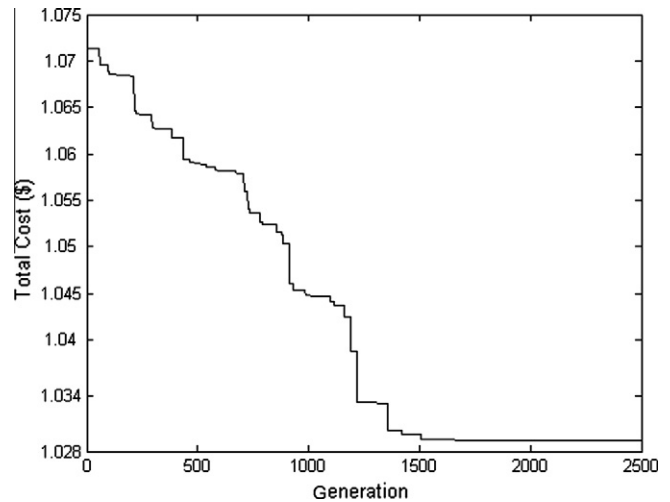


- Probability of mutation: 0.01.
- Full elitism.
- Population size: 50.

The results obtained with DACGA for the different systems, and compared with the results obtained by other authors; show that it was possible to find better solutions (Table 1).

**Table 1**  
Comparison with other results.

	Total cost (\$)	Convergence (generations)
<i>10-Unit system</i>		
ICGA (Damousis et al., 2004)	566,404	300
MRCGA (Sun et al., 2006)	564,244	500
FPGA (Dang & Li, 2007)	564,094	>3000
DACGA	563,987	<300
<i>38-Unit system</i>		
MRCGA (Sun et al., 2006)	206,000	1900
DACGA	195,042	<1500
<i>45-Unit system</i>		
PRGA (Arroyo & Conejo, 2002)	1,029,557	200
DACGA	1,029,100	<2000



**Fig. 15.** Convergence for the 45-unit system.

Convergence on the best solution is obtained in a smaller number of generations as compared to MRCGA (Sun et al., 2006) and FPGA (Dang & Li, 2007). Fig. 13 shows the convergence for the 10-unit system. When compared with ICGA (Damousis et al., 2004) a similar convergence is observed, but it must be considered that ICGA proposes a chromosome representation with real coding which reduces the size of the chromosome.

Fig. 14 shows the convergence for the 38-unit system, for which it is observed that below 1500 generations a lower cost can be obtained as compared to that obtained by MRCGA (Sun et al., 2006).

Finally, the convergence in the 45-unit system is shown in Fig. 15. In the case of PRGA (Arroyo & Conejo, 2002) the convergence cannot be compared directly, since a parallel optimisation technique is proposed which works with up to 32 processors.

## 6. Conclusions

The results found, when compared with those obtained by other GA, validate the application of DACGA to the unit commitment problem. An improvement is observed in the convergence and in the quality of the best solution found.

Thanks to the deterministic selection operator, a greater diversity between the individuals of the population can be obtained with this proposal because it uses the genetic information of the worst individuals with the same probability, thus complementing the characteristics of the best individuals of the population.

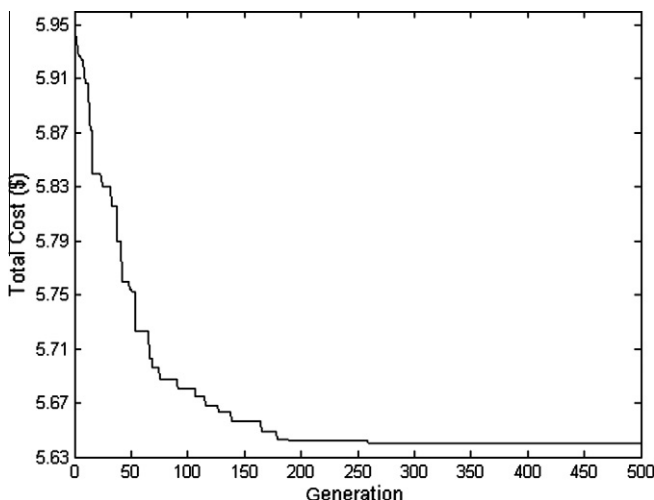
With ring representation, the start time is contiguous with the finish time of the planning period. Thus the proposed annular crossover operator allows a greater probability of exchange of genetic information between the start and finish periods of the 24 h planning.

## Acknowledgements

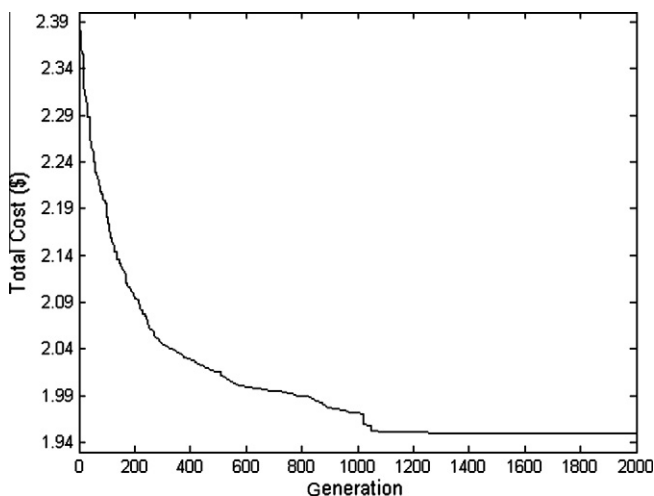
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**Fig. 13.** Convergence for the 10-unit system.



**Fig. 14.** Convergence for the 38-unit system.

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