Distributed Consensus-Based Economic Dispatch With Transmission Losses

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***Abstract*—A distributed algorithm is presented to solve the economic power dispatch with transmission line losses and generator constraints. The proposed approach is based on two consensus algorithms running in parallel. The first algorithm is a first-order consensus protocol modified by a correction term which uses a local estimation of the system power mismatch to ensure the generation-demand equality. The second algorithm performs the estimation of the power mismatch in the system using a consensus strategy called consensus on the most up-to-date information. The proposed approach can handle networks of different size and topology using the information about the number of nodes which is also evaluated in a distributed fashion. Simulations performed on standard test cases demonstrate the effectiveness of the proposed approach for both small and large systems.**

***Index Terms*—Consensus protocols, economic dispatch, multi-agent systems, smart grid.**

I. Introduction

THE emerging smart grid framework helps revisiting some of the fundamental challenges in power distribution systems, e.g., the economic dispatch (ED). The ED distributes the total power demand among the generating units, while minimizing the operating cost and satisfying both unit- and system-level constraints [1]. Traditional optimization techniques [1] such as lambda iteration method [2], gradient method, linear programming and Newton's method can be sensitive to starting points and could converge to a local optimum or diverge altogether [3]. Conventional approaches also include heuristic methods such as genetic algorithm [4] and particle swarm optimization [5], that can handle non-convex solution spaces and more stringent constraints. Moreover, recent advancements account for emission dispatch [6] and intermittency of renewable energy sources [7]. Despite excellent performance, majority of existing ED approaches are performed centrally.

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As the physical power system is moving from the legacy grid with a classic centralized structure to a smart grid with a more distributed nature, a similar paradigm shift takes place in the decision-making and cyber domains [8]. A centralized controller usually requires high bandwidth communication infrastructure to act on system-wide gathered information, needs a high-level of connectivity, poses reliability concerns due to the presence of a single point-of-failure, and is prone to modeling error. Moreover, both the future power grid and the communication network are likely to have a variable topology, which undermines the efficacy of a centralized mechanism. Alternatively, distributed algorithms can be more suitable to handle topological variations and accommodate plug-and-play features. Moreover, they are more robust, scalable, and can better accommodate a large number of units compared to centralized approaches. Finally, a distributed decision-making tool can effectively utilize sparse communication infrastructures with limited message passing among participating units.

The ED can be formulated as a constrained optimization problem characterized by a Lagrangian variable, namely the incremental cost. Thus, consensus-based approaches are particularly appropriate since ED implies equal incremental costs for all generating units. Neglecting the transmission losses, [9]–[12] propose consensus algorithms on the incremental cost where the power mismatch between generation and demand is directly considered in the algorithm formulation [9], assumed known a priori by a leader node [10], found in a distributed fashion using additional level of consensus [11], or found using an innovation term in addition to the consensus term [12]. A double-iteration algorithm is proposed in [13], considering lower and upper bounds on the allocated resources. In [14] a distributed iterative procedure is presented for the load-frequency control and economic dispatch that requires adjusting some parameters with an initial centralized intervention. The cited distributed solutions are developed for loss-less scenarios, although the transmission losses are key elements for accurate formulation of the ED problem.

In this paper, a distributed solution is provided to solve the ED problem considering transmission losses as well as generation limits. The proposed approach is based on a consensus procedure with a correction term to satisfy the generation-demand equality constraint. The distributed estimation of the global power mismatch in the system is simultaneously performed by each bus with a protocol, referred to as consensus on the most up-to-date information, that is inspired by task allocation in multi-robot systems [15]. Such protocol lets each node on the communication graph to save the most up-to-date information from the other nodes by using timestamp information associated to the state value. The salient features of the proposed algorithm are the following:

- The economic dispatch problem is solved in a distributed fashion, including the transmission losses which are computed locally at each bus;
- The estimation of the global power mismatch does not require additional level of consensus, i.e., all the consensus protocols run in parallel, resulting in a faster procedure;
- The proposed consensus protocol relies on a correction term that can be defined as a function of the number of nodes, effectively handling networks of different sizes without requiring manual intervention on global convergence parameters.

The rest of this paper is organized as follows. Section II discusses the ED problem and existing centralized solutions. Section III provides an overview of graph theory and consensus protocols. The proposed distributed solution is presented in Section IV. Several case studies are presented in Section V to show the effectiveness of the proposed approach. Section VI concludes the paper.

II. Problem Formulation and Centralized Solution for Economic Dispatch

In this section, the ED problem with transmission losses and generator constraints is discussed, and classical centralized solutions are introduced.

A. Economic Dispatch Problem

The goal of ED is to minimize the power generation cost given by

$$f(p_G) = \sum_{i \in \mathcal{S}_G} f_i(p_{Gi}) \tag{1}$$

where $\mathcal{S}_G$ is the set of generator buses, p_{Gi} is the generated power at bus $i \in \mathcal{S}_G$, $f_i(p_{Gi})$ is the cost function for generation at bus i, and $p_G = [p_{G1}, p_{G2}, \ldots, p_{Gn_G}]^T$ denotes the power vector for all generating buses, with $n_G = |\mathcal{S}_G|$ the number of generating buses. The generation cost function $f_i(p_{Gi})$ is usually approximated [1] by a quadratic function

$$f_i(p_{Gi}) = \alpha_i + \beta_i p_{Gi} + \gamma_i p_{Gi}^2 \tag{2}$$

where α_i, β_i, and γ_i are the cost coefficients of unit i.

The ED problem is subject to several operational constraints. Denoting by p_{Di} the load demand at bus i, the system load demand is $p_D = \sum_{i \in \mathcal{S}_D}(p_{Di})$, with $\mathcal{S}_D$ as the set of load buses and $n_D = |\mathcal{S}_D|$ as the number of load buses. Then, the generation-demand equality constraint

$$\Delta P = \sum_{i \in \mathcal{S}_D} p_{Di} + p_L - \sum_{i \in \mathcal{S}_G} p_{Gi} \tag{3}$$

states that the total generated power $\sum_{i \in \mathcal{S}_G}(p_{Gi})$ should be equal to the system load demand, p_D, plus the transmission network losses, p_L. The variable ΔP denotes the global power mismatch. The losses can be expressed by

$$p_L = \sum_{(i,j) \in \mathcal{S}_B \times \mathcal{S}_B} R_{ij} I_{ij}^2 / 2 \tag{4}$$

where $\mathcal{S}_B$ is the set of buses, R_{ij} is the resistance of the line connecting buses i and j, and I_{ij} is the current in the transmission line between buses i and j (with $I_{ij} = 0$ if there is no direct connection between the two buses).

The power generator constraints given by

$$p_{Gi}^{\min} \le p_{Gi} \le p_{Gi}^{\max} \quad i = 1, \ldots, n_G \tag{5}$$

state that the generated power p_{Gi} cannot exceed the corresponding minimum, $p_{Gi}^{\min}$, and maximum, $p_{Gi}^{\max}$, bounds.

The ED is conventionally formulated using the Lagrangian operator [16] such that

$$L(p_G, \lambda) = f(p_G) + \lambda \Delta P + \sum_{i=1}^{2n_G} \mu_i \varphi_i \tag{6}$$

where λ and μ_i are the Lagrange multipliers associated with the equality constraint in (3) and the power generator constraints in (5), respectively. Then, the Lagrangian operator L is minimized with respect to the generated power, while the constraints are satisfied.

B. Centralized Solution

A key concept of centralized solutions for ED is the incremental cost, that is, the difference in costs as a result of adding/subtracting one unit of power. The incremental cost, therefore, is the derivative of the cost function with respect to the power. When generator constraints and transmission losses are neglected, the Lagrangian operator provides a set of n equations

$$\frac{\partial L}{\partial p_{Gi}} = \frac{df_i(p_{Gi})}{dp_{Gi}} - \lambda = 0 \tag{7}$$

or, equivalently

$$\lambda = \frac{df_i(p_{Gi})}{dp_{Gi}}. \tag{8}$$

Therefore, the necessary condition for the existence of a minimum-cost operating point is that all incremental costs must be equal to λ. In this case, the optimal incremental cost $\lambda^\star$ and the generated power $p_{Gi}^\star$ are given [9] by

$$\lambda^\star = \left(p_D + \sum_{i \in \mathcal{S}_G} \frac{\beta_i}{2\gamma_i} \right) \Bigg/ \left(\sum_{i \in \mathcal{S}_G} \frac{1}{2\gamma_i} \right) \tag{9}$$

$$p_{Gi}^\star = (\lambda^\star - \beta_i)/(2\gamma_i). \tag{10}$$

When the generator constraints are considered, the necessary conditions for the existence of a minimum-cost operating condition may be expanded slightly as in [1]

$$\begin{cases} \frac{df_i(p_{Gi})}{dp_{Gi}} = \lambda & p_{Gi}^{\min} \le p_{Gi} \le p_{Gi}^{\max} \\ \frac{df_i(p_{Gi})}{dp_{Gi}} \le \lambda & p_{Gi} = p_{Gi}^{\max} \\ \frac{df_i(p_{Gi})}{dp_{Gi}} \ge \lambda & p_{Gi} = p_{Gi}^{\min}. \end{cases} \tag{11}$$

This implies that all the units with non-active bounds have the same incremental cost, while other units have incremental cost $(\beta_i + 2\gamma_i p_{Gi}^{\min})$ or $(\beta_i + 2\gamma_i p_{Gi}^{\max})$ when operating on the limits $p_{Gi}^{\min}$ or $p_{Gi}^{\max}$, respectively.

When the transmission losses are considered, the Lagrangian operator gives a set of n equations

$$\frac{\partial L}{\partial p_{Gi}} = \frac{df_i(p_{Gi})}{dp_{Gi}} - \lambda\left(1 - \frac{\partial p_L}{\partial p_{Gi}}\right) = 0 \tag{12}$$

or, equivalently

$$\lambda = \frac{1}{\left(1 - \frac{\partial p_L}{\partial p_{Gi}}\right)} \frac{df_i(p_{Gi})}{dp_{Gi}} \tag{13}$$

where $(\partial p_L)/(\partial p_{Gi})$ and $Pf_i = 1/(1 - (\partial p_L)/(\partial p_{Gi}))$ are called *incremental loss* and *penalty factor* for unit i, respectively. The ED problem with transmission losses can be solved by making the incremental cost at each unit the same, but the resulting p_{Gi} values will be somewhat different from the ideal dispatch scenario (depending on Pf_i and $(df_i(p_{Gi}))/(dp_{Gi})$). In this scenario, the ED can become analytically intractable even with simplified cost functions [14].

III. Graph Theory and Consensus Protocols

In this section, notations of graph theory are presented to model the power system, and the basic first-order consensus protocol is introduced.

A. Graph Theory

A graph G is used to model the power system elements (i.e., buses and transmission lines) and the way such elements can exchange information using a communication infrastructure. It is defined as $G = (\mathcal{V}, \mathcal{E}, A)$, where $\mathcal{V} = \{\nu_1, \nu_2, \ldots, \nu_{n_B}\}$ is a set of elements called nodes, $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is a set of pairs of distinct nodes called edges, and $A = [a_{ij}] \in R^{n_B \times n_B}$ is the associated adjacency matrix. A directed graph is a graph where the edges have directions associated with them. Graph nodes represent the buses of the power system, the edges represent the transmission lines among the buses, and the adjacency matrix reflects the edge weights. When bus i can receive information from bus j, an edge denoted by (v_j, v_i) exists from bus j to bus i with weight a_{ij}. The element a_{ij} of the adjacency matrix is positive only if the corresponding edge exists; otherwise, it is zero. Bus j is called neighbor of bus i if bus i can get information from bus j. The set of neighbors for bus i is denoted by $\mathcal{N}_i$ with cardinality $\bar{d}_i$. The volume is defined as $v = \sum_{i=1}^n \bar{d}_i$. A path from bus i to bus j is a sequence of edges connecting the two buses. The distance d_{ij} between buses i and j is the number of edges in the shortest path connecting them, while the diameter ϕ of the graph is the greatest minimum distance between any pair of buses.

B. First-Order Consensus Protocols

Suppose that each bus i carries a state $x_i \in R$ representing a physical quantity such as the output power, incremental cost, power mismatch, etc. Assume that the communications between buses can be performed at discrete time instants denoted by k, the discrete time sample. Thus, a bus dynamics can be described by the discrete-time equation

$$x_i(k+1) = x_i(k) + u_i(k) \tag{14}$$

that specifies how the carried information evolves by time.

In the consensus problem [17], [18], all nodes reach the same state, that is

$$x_i(k) - x_j(k) \to 0 \quad \forall (i,j). \tag{15}$$

Assuming that A is row stochastic (i.e., row sum of 1) and $a_{ii} > 0$ for all i, the most common protocol [17], [18], has the form

$$u_i(k) = \sum_{j \in \mathcal{N}_i} a_{ij}(x_j(k) - x_i(k)) \tag{16}$$

with a_{ij} the graph edge weights and gives the corresponding dynamic

$$x_i(k+1) = \sum_{j \in \mathcal{N}_i} a_{ij} x_j(k). \tag{17}$$

Intuitively, the state of each node is updated as the weighted average of its current state and the current states of its neighbors.

Under protocol (16), if the communication graph is connected, each node reaches the final value [17], [18] given by

$$x_c = \sum_{i=1}^{n} \eta_i x_i(0) \tag{18}$$

with $w_1 = [\eta_1, \ldots, \eta_n]$ being the normalized left eigenvector of the adjacency matrix A associated to the eigenvalue 1. This means that the consensus value is the linear combination of the initial states weighted by the elements of the normalized left eigenvector of A associated to the eigenvalue 1.

IV. Distributed Solution for the ED

The proposed approach solves the ED problem in a distributed fashion by using consensus protocols based on communications between neighbors on the communication graph supporting the power system.

A. Distributed Approach

Fig. 1 shows the flowchart of the proposed algorithm, referred to as *lambda-consensus*. Two consensus protocols are run in parallel: the first one is used to reach consensus on the Lagrange multiplier, while the second one is used to estimate the power mismatch in the system.

The rationale of the proposed approach is to reach consensus on the Lagrangian multiplier by running the following protocol

$$\begin{aligned} \lambda_i(k+1) &= \sum_{j \in \mathcal{N}_i} a_{ij}\lambda_j(k) + K_P 2\gamma_i \Delta P_i(k) \quad \forall i \in S_G \\ \lambda_i(k+1) &= \sum_{j \in \mathcal{N}_i} a_{ij}\lambda_j(k) \quad \forall i \in S_D \end{aligned} \tag{19}$$

where $\Delta P_i(k)$ is the global power mismatch evaluated by bus i, including the transmission losses, and K_P is a proportional con-

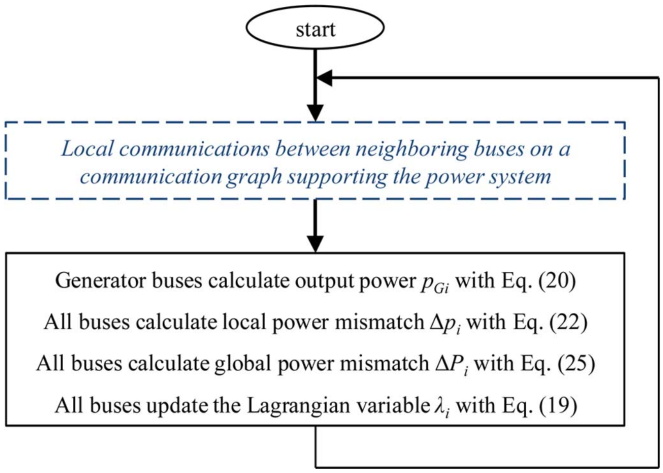

Fig. 1. Flowchart of the proposed lambda-consensus algorithm.

trol gain. Then, each generator bus locally compute its output power, fulfilling the generator constraints, as

$$p_{Gi}(k) = \begin{cases} \frac{\lambda_i - \beta_i}{2\gamma_i} & p_{Gi}^{\min} \le \frac{\lambda_i - \beta_i}{2\gamma_i} \le p_{Gi}^{\max} \\ p_{Gi}^{\min} & \frac{\lambda_i - \beta_i}{2\gamma_i} \le p_{Gi}^{\min} \\ p_{Gi}^{\max} & \frac{\lambda_i - \beta_i}{2\gamma_i} \ge p_{Gi}^{\max}. \end{cases} \tag{20}$$

Each generator bus runs a consensus protocol modified with a correction term $K_P 2\gamma_i \Delta P_i(k)$ to find the unknown value of the Lagrangian multiplier. Each load bus runs a consensus protocol without the correction term to propagate the Lagrangian multiplier information among the network. The correction term acts as a distributed proportional controller that let each generator bus change the incremental cost to satisfy the generation-demand equality constraint. That is, each generator bus increases (or decreases) the incremental cost and generates more (or less) power when the power mismatch $\Delta P_i(k)$ is positive (or negative). The correction action is proportional to the gain K_P and the term $2\gamma_i$. The gain K_P regulates the respective convergence speeds of consensus and correction operations (more details are given in Section IV-D), while the term $2\gamma_i$ derives by the practical proportionality between power mismatch and incremental cost. In fact, given a power p_{Gi}, the corresponding incremental cost is $\lambda_i = \beta_i + 2\gamma_i p_{Gi}$. Considering an increased power $p_{Gi} + \Delta p_{Gi}$, one has $\lambda_i + \Delta\lambda_i = \beta_i + 2\gamma_i(p_{Gi} + \Delta p_{Gi})$, that is $\Delta\lambda_i = (\beta_i + 2\gamma_i p_{Gi}) - \lambda_i + 2\gamma_i \Delta p_{Gi} = 2\gamma_i \Delta p_{Gi}$. Therefore, λ_i changes proportionally to $2\gamma_i$ as a result of the power mismatch Δp_{Gi}.

B. Power Mismatch

The modified consensus protocol in (19) requires that each bus cooperates to estimate the global power mismatch in the network. This operation can be performed by using a consensus idea, referred to as *consensus on the most up-to-date information*. The basic idea is to associate timestamp information to the state value. Then, each bus can store the most up-to-date information received by other buses to estimate the required global information.

Assume that each bus i knows the resistance R_{ij} for the transmission lines with its neighbors $j \in \mathcal{N}_i$. Then, it can locally compute a local portion of the losses in the power system as

$$p_{Li} = \sum_{j \in \mathcal{N}_i} R_{ij} I_{ij}^2 / 2 \tag{21}$$

with I_{ij} the current in the transmission line between buses i and j that, in practical implementations, can be obtained by the protective relay devices [19], [20]. Thus, each bus can effectively compute locally the associated losses. Moreover, since the generated power p_{Gi} and the load demand p_{Di} are also locally known, each bus can compute its local power mismatch as

$$\Delta p_i = p_{Di} + p_{Li} - p_{Gi}. \tag{22}$$

It is straightforward that the sum of the local power mismatch Δp_i is equal to the global power mismatch ΔP.

Now suppose that each bus carries a state vector $x_i \in R^{n_B}$ and a timestamp vector $s_i \in R^{n_B}$. x_{ij} and s_{ij} are the jth element of x_i and s_i, respectively. The element x_{ii} carries the information about the local power mismatch at bus i evaluated by bus i, while the element x_{ij} carries the information about the local power mismatch at bus j as known by bus i. Such values are updated with the most up-to-date information received from the neighbors by using the timestamp s_{ij} associated to x_{ij}. In details, each bus runs the following protocol:

$$\begin{aligned} s_{ii}(k+1) &= k+1 \\ x_{ii}(k+1) &= \Delta p_i(k) \\ s_{ij}(k+1) &= \max_{h \in \mathcal{N}_i}(s_{hj}(k)) \quad i \ne j \\ x_{ij}(k+1) &= x_{i_j^\star j}(k) \quad i \ne j \end{aligned} \tag{23}$$

with

$$i_j^\star = \arg\max_{h \in \mathcal{N}_i}(s_{hj}(k)). \tag{24}$$

These dynamics let each bus to build the vector $x_i(k)$ storing the most up-to-date local power mismatches Δp_i's for all the buses. Thus, each generator bus i can estimate the global power mismatch in the power system as

$$\Delta P_i(k) = \sum_{j \in \mathcal{S}_B} x_{ij}(k). \tag{25}$$

Each generator bus i updates $\lambda_i(k)$ according to its knowledge of the global power mismatch $\Delta P_i(k)$. As the algorithm proceeds, all the buses reach consensus on $x_i(k)$, and $\lambda_i(k)$ is updated until the global power mismatch disappears.

C. Number of Nodes

The information about the number of nodes in the system can be used to define the control gain K_P as proposed in the following section. Thus, an approach for its distributed computation is presented here to make the proposed solution for the ED fully distributed.

The consensus on the most up-to-date information presented in Section IV-B inherently contains the information about the number of nodes. Such a value is equal to the size of the time-stamp and state vectors which are built dynamically. Thus, the idea is to exploit this feature of the consensus on the most up-to-date information to dynamically update the information about the number of nodes.

The implementation of protocol (23) expects each message to be marked with a unique identifier (ID). Thus, a message from node i can be represented by a tuple $m_i = \langle id_i, s_i, x_i \rangle$, where id_i is the ID vector and has the same size of s_i and x_i. Consider the illustrative example in Fig. 2. At the beginning, as shown in Fig. 2(a), since no node has any information about other nodes, the node number estimation is $n_i = 1$ for all the nodes. After a first iteration, as shown in Fig. 2(b), each node i receives messages from its neighbors. After updates (23)–(24), it results in $id_i, s_i, x_i \in R^{1+|\mathcal{N}_i|}$ and the node number estimation $n_i = 1+|\mathcal{N}_i|$. For example, node 1 receives a message only from node 2 and set $n_1 = 2$, while node 2 receives messages from both node 1 and 3 and set $n_2 = 3$. With the same mechanism, after a second iteration, as shown in Fig. 2(b), each node i knows about the presence of all nodes j with distance $d_{ij} = 2$, and so on. This means that each node knows the total number of nodes in the network within a number of iterations equal to the network diameter, ϕ. In the example shown in Fig. 2, the network diameter is $\phi = 3$ and all the nodes compute $n_i = 4$ within 3 iterations.

Moreover, the information carried by the timestamp vector can be exploited to understand when a node is no longer in the network. In fact, if a node i does not receive any updates about node j within a number of iterations equal to ϕ (that is, the timestamp is not updated for at least ϕ iterations), then node j can be removed from local information of node i. Therefore, considering that an upper bound for the diameter is $n-1$, in the case of a line network as shown in Fig. 2(a), it is possible to use this upper bound as a threshold to consider a node out of the network.

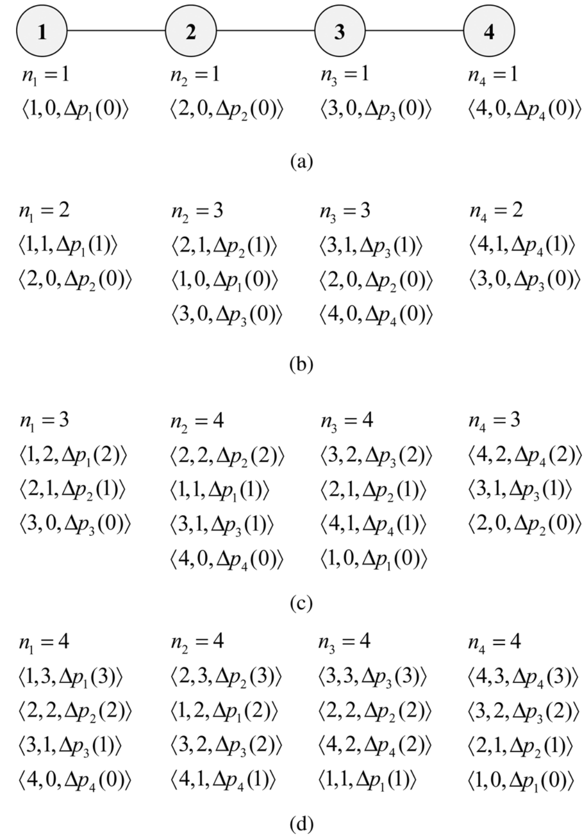

Fig. 2. Description of message passing and distributed computation of the number of nodes. Each message available locally is represented by a tuple $\langle i, s_i, \Delta p_i \rangle$, with i as the bus identifier, s as the timestamp of the message, and Δp as the local power mismatch for bus i. (a) Local information at time step $k = 0$. (b) Local information at time step $k = 1$. (c) Local information at time step $k = 2$. (d) Local information at time step $k = 3$.

D. Proportional Gain Design

The proposed distributed solution is based on the consensus procedure in (19) and the estimation of the power mismatch in (25). The choice of the control gain K_P in (19) affects both the convergence speed and the algorithm response since the two consensus protocols are run in parallel.

Considering the consensus procedure in (19), the smaller the control gain K_P, the smaller the correction applied by each generator bus to the corresponding λ_i, and the slower the algorithm convergence. Therefore, from this viewpoint, larger values of the control gain K_P would lead to a faster convergence. On the other hand, K_P should not be set too large to avoid excessively fast updates of the local λ_i's which could lead to oscillatory behavior. Moreover, the consensus procedure in (19) must be faster than the tendency of the correction term $K_P 2\gamma_i \Delta P_i(k)$ to modify various λ_i's to reach consensus. This suggests keeping the value of K_P sufficiently small to guarantee effective convergence of the algorithm.

Moreover, the network size affects the choice of the control gain K_P. One can observe that, given a certain K_P, the larger the network, the slower the convergence rate for the consensus algorithm in (19). Furthermore, the larger the network diameter, the slower the information propagation for the consensus on the most up-to-date information. This implies that smaller networks can use larger values of K_P without causing the undesired behavior described above. This suggests defining a design criterion for the control gain K_P as a function of the topological properties of the communication graph that could be used to adjust K_P on-line. A possible heuristic candidate is

$$K_P = 1/n_B \tag{26}$$

where n_B is the number of buses in the power system. Therefore, the larger the system, the smaller the control gain K_P and the correction term computed in (19) for each generator bus. It should be noted that each bus can now locally compute K_P since the number of buses is known locally, as described in Section IV-C. This heuristic choice will be shown in Section V to be an effective choice for both small and large scenarios.

E. Plug-and-Play Implementation

The consensus protocol in (19) requires the definition of the adjacency matrix A that has to be row stochastic (i.e., row sum of 1), as described in Section III-B. Practically, each bus i must

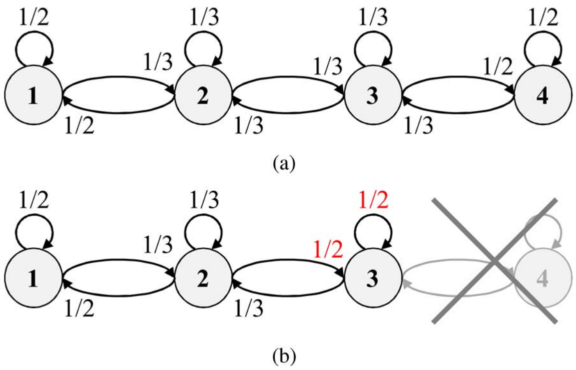

Fig. 3. Adjacency matrix for the communication graph according to the bidirectional equal-neighbor weight definition: (a) initial configuration of the network with 4 buses, and (b) automatic updating of the weights when bus 4 is out of the network.

locally know the i-th row of the matrix to perform the consensus operation. Since the algorithm has to accommodate plug-and-play features, each node should evaluate locally its corresponding row in the adjacency matrix and should be able to update online such a row when the topology changes by using only the local information.

Assume that the communication graph among the buses satisfies the bidirectional equal-neighbor weight topology [21], i.e.,

$$(\nu_i, \nu_j) \in \mathcal{E} \Rightarrow (\nu_j, \nu_i) \in \mathcal{E}, \quad (\nu_i, \nu_i) \in \mathcal{E} \tag{27}$$

with the elements a_{ij} of the adjacency matrix defined as

$$a_{ij} = \begin{cases} 1/\bar{d}_i & \text{if } j \in \mathcal{N}_i \\ 0 & \text{if } j \notin \mathcal{N}_i \end{cases} \tag{28}$$

where $\mathcal{N}_i$ is the set of neighbors (including the same bus i) and $\bar{d}_i$ is its cardinality. Such a definition leads to a row stochastic adjacency matrix as required. Moreover, each bus can locally build its corresponding row in the adjacency matrix by simply counting the number of neighbors. Since each bus can understand when nodes are added or removed with the mechanism described in the previous section, each node can update online its corresponding row of the adjacency matrix and reach consensus. An example is given in Fig. 3.

V. Simulation Results

In this section, several case studies are discussed to show the effectiveness of the proposed lambda-consensus algorithm. The first two case studies show the algorithm performance with and without power generator constraints. The third case study considers a time-varying load demand, while the fourth one demonstrates the plug and play properties of the proposed algorithm. Then, the IEEE 300-bus system is used to validate the algorithm on a large case. Finally, the comparison with the lambda-iteration method shows the effectiveness of the proposed lambda-consensus algorithm.

A. Case Study 1: Without Generator Constraints

The power system with 6 buses, 3 generators, and 11 lines from [1] is considered. The single-line diagram of the power

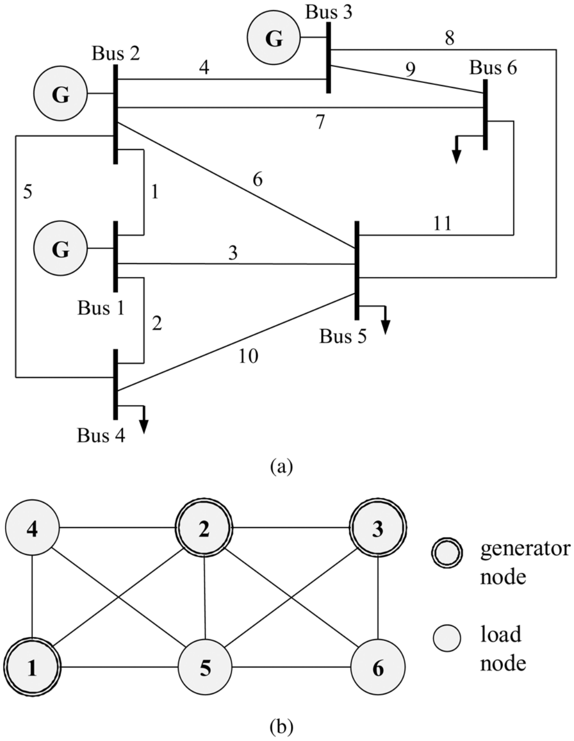

Fig. 4. 6-bus system: (a) single-line diagram, and (b) corresponding communication graph.

TABLE I
Generator Characteristics of the 6-Bus System in [1]

Generator bus	α_i [$/MW^2$]	β_i [$/MW$]	γ_i [$]	p_i^{min} [MW]	p_i^{max} [MW]
1	213.1	11.669	0.00533	50	200
2	200	10.333	0.00889	37.5	150
3	240	10.833	0.00741	45	180

system is shown in Fig. 4(a), while the corresponding communication graph is shown in Fig. 4(b). The characteristics of the generators are given in Tables I, while the resistances for the transmission lines are given in the archive of Matpower [22]. The simulations have been run in Matlab and the current values for the losses computation have been estimated by running the power flow routine of Matpower [22]. For convenience of simulation, the losses are computed using the initial condition of the system and then are kept constant during the simulation. This does not impact the validity and completeness of the presented results since time-varying losses can be considered as time-varying demands from the viewpoint of the algorithm and, in Section V-C, the algorithm will be shown to properly work in such a condition.

In this case study, the power generator constraints are not considered. The initial load demand is 210 MW, with three loads of 70 MW for each load bus. At the time step $k = 10$, all loads are doubled, so the total load demand is 420 MW. The system responds automatically to the change in the load demand converging to the new solution as shown in Fig. 5. The Lagrange multiplier is increased according to the estimation of the system power mismatch and the consensus is reached after a few iterations. The power mismatch goes to zero and the sum of the output powers satisfies the generation-demand equality

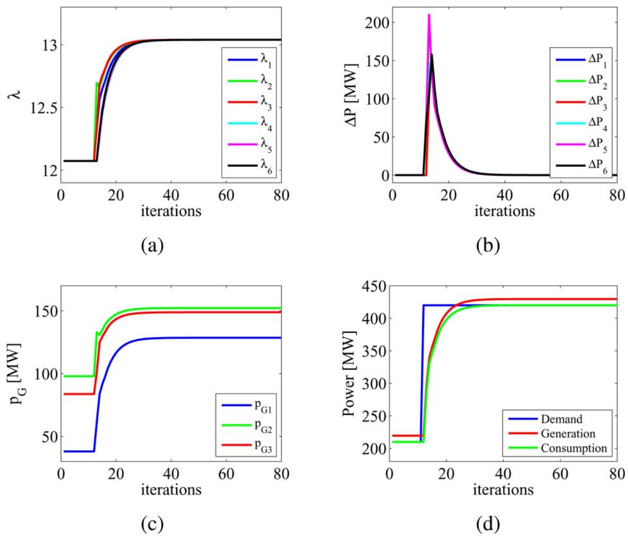

Fig. 5. Case study 1, without power generator constraints: (a) Lagrange multiplier, (b) power mismatch, (c) output power, (d) demand, generation and consumption.

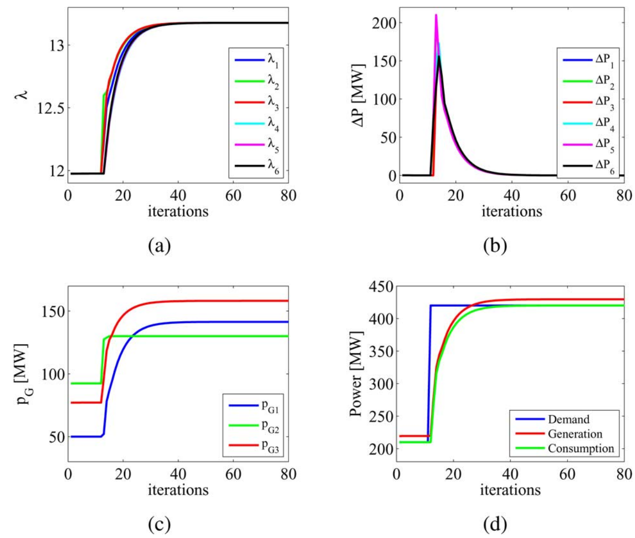

Fig. 6. Case study 2, with power generator constraints (imposing a maximum power generator $p_2^{\max} = 130$ MW to better visualize the algorithm behavior): (a) Lagrange multiplier, (b) power mismatch, (c) output power, (d) demand, generation and consumption.

constraint including the losses. The generator output powers are $p_1 = 127.33$ MW, $p_2 = 151.48$ MW, and $p_3 = 148.00$ MW, with $\lambda = 13.03\$/\text{MW}$. Note that the second generator is not fulfilling the constraint on its maximum output power $p_2^{\max} = 150$ MW. The next case study will consider also the power generator constraints.

B. Case Study 2: With Generator Constraints

This case study considers the power system of case study 1 adding the power generator constraints. A maximum power generator constraint $p_2^{\max} = 130$ MW is forced on the second generator to better visualize in Fig. 6 the behavior of the proposed algorithm. The generator output powers are $p_1 = 139.82$ MW, $p_2 = 130.00$ MW, and $p_3 = 156.99$ MW, with $\lambda = 13.16\$/\text{MW}$. In this case, the second generator is not exceeding its maximum output power and the other two generators have to increase their output powers with respect to the previous case to supply for the saturation of the second generator. Thus, the system converges to the new solution reaching consensus in a few iterations also when considering the power generator constraints.

C. Case Study 3: Time-Varying Demand

In this case study, the focus is on the capability of the proposed algorithm to handle automatically changes in the load demand. Considering the usual example with 6 buses as in case study 1, the load demand is changed 3 times during the simulation, as shown in Fig. 7. At the time step $k = 10$, the system load demand is increased by 40%, while at the time step $k = 60$ it is reduced by 20%. Finally, at the time step $k = 110$, the system load demand is increased again by 50%. The system responds automatically to each change in the load demand converging to a new solution as shown in Fig. 7. The Lagrange multiplier is modified according to the new demand, the power mismatch goes to zero and the sum of the output powers satisfies the generation-demand equality constraint including the

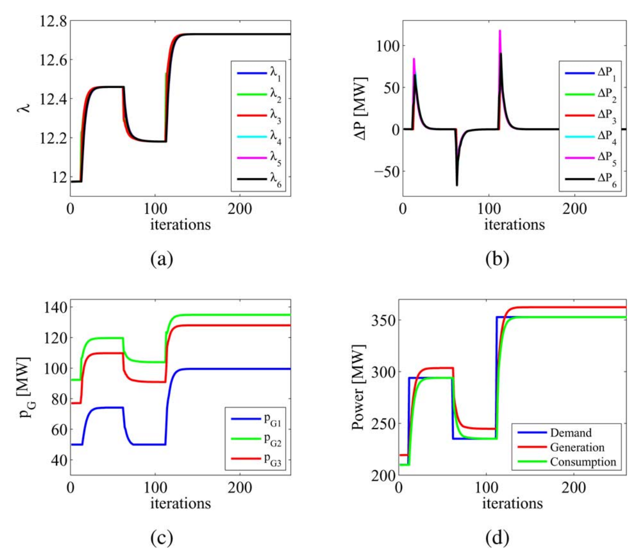

Fig. 7. Case study 3, time-varying demand: (a) Lagrange multiplier, (b) power mismatch, (c) output power, (d) demand, generation and consumption.

losses. Thus, the lambda-consensus algorithm works properly under time-varying loading conditions.

D. Case Study 4: Plug and Play Capability

In this case study, the plug and play adaptability of the proposed lambda-consensus is discussed considering the same power system as in case study 1. This simulation considers an initial output distribution with all the generator buses providing their minimum output power with the corresponding local incremental cost. At the beginning, individual buses do not have information about the power system and set their bus number variable as $n_i(0) = 1$. Fig. 8 shows that each bus can obtain the total number of buses in the whole system within a few iterations by using the described mechanism in Section IV-C. The system converges to a steady condition providing the system load demand of 210 MW. At the time step $k = 70$, the

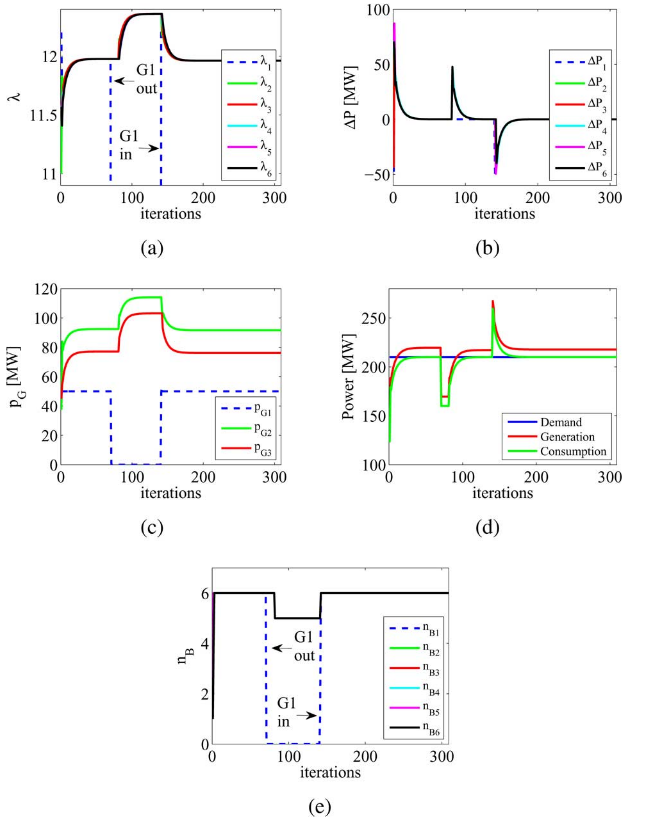

Fig. 8. Case study 4, plug and play capability: (a) Lagrange multiplier, (b) power mismatch, (c) output power, (d) demand, generation and consumption, and (e) number of buses. Generator bus 1 (blue dash line) is first removed at the time step 70, and then added again at the time step 140.

generator bus 1 is removed from the system. The remaining buses detect the disconnection of this generator, decrease the bus number variable, consider the power mismatch under the new conditions and converge to a new solution. Of course, the remaining generators have to generate more power to compensate for the amount of power previously generated by the disconnected generator. At the time step $k = 140$ the generator bus 1 is connected again and the system properly responds to this new topological change. The buses detect the presence of an additional generator and the whole system reaches the same solution prior to disconnection. Thus, this case study clearly shows the plug and play capability of the proposed distributed approach.

E. Case Study 5: Implementation on IEEE 300-Bus System

In this case study, the IEEE 300-bus system with 69 generator buses and 411 transmission lines [22] is considered to show the effectiveness of the proposed approach for a large network. The simulation starts with the initial condition given by the Matpower case study [22] and a system load demand of 23 526 MW. Then, at the time steps $k = 400$ and $k = 800$, the system load demand is first increased by 20% and then reduced by 10%, respectively. As shown in Fig. 9, the lambda-consensus algorithm converges automatically to a new solution, fulfilling the power generator constraints and the generation-demand equality constraint, including the transmission losses. Moreover, each iteration requires about 0.007 seconds per bus running on a Personal

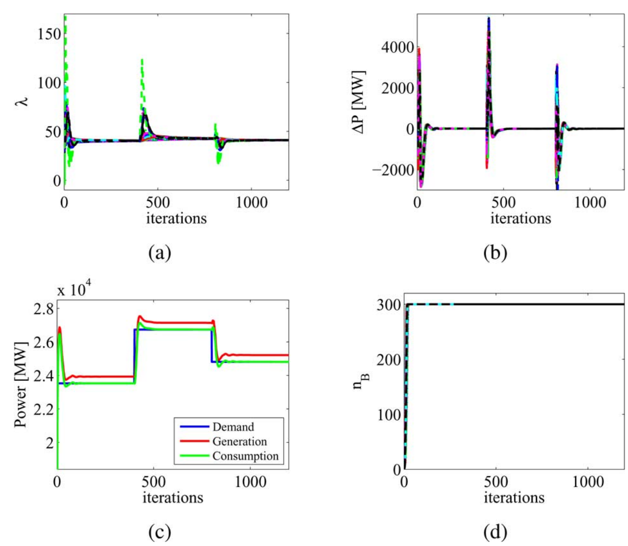

Fig. 9. Case study 5, IEEE 300-bus system: (a) Lagrange multiplier, (b) power mismatch, (c) demand, generation and consumption, and (d) number of buses.

Computer with a Core2 Duo processor (3 GHz) and 4 GB RAM. Since the algorithm converges in a few hundred iterations, assuming a conservative time step of 0.02 seconds for a practical application, the algorithm converges in a few seconds. Thus, the algorithm is sufficiently fast and suitable for actual implementation considering that the deployment time for the economic dispatch solution is usually from 5 to 15 min [23].

F. Case Study 6: Comparison With Lambda-Iteration Method

In this section, the performance of the proposed lambda-consensus method are compared with the well-known lambda-iteration method [2], [1]. The original algorithm in [2] does not include the losses, while in [1, p. 118], a version considering the losses in terms of B matrix is presented. To perform a meaningful comparison with this second variant including the transmission losses, the following methodology has been adopted. First, note that the proposed algorithm can properly work given a generic function to compute the losses. The loss function in (21) is adopted due to its inherently distributed nature, but any function computing the losses based on different quantities leads to the same results. Second, the B matrix representation defines the losses as a function of the generated output powers, thus it involves only the generator buses and includes in the B coefficients the knowledge of power network topology. In order to adapt this loss representation *only* for simulation purposes, each generator bus can substitute (21) with $p_{Li}(k) = x_{ii}(k)\sum_{i\in\mathcal{S}_G} B_{ij}x_{ij}(k) + x_{ii}(k)B_{0,i} + B_{00}/n_G$, while each load bus computes a zero value for its local losses. It is worth noting that the sum of the losses within the power system is independent of the way they are computed within the approximation of each representation. In this way, it is possible to perform a fair and meaningful numerical comparison between the two algorithms.

Tables II, III and IV compare the proposed lambda-consensus algorithm with the lambda iteration method handling the transmission losses. The 6- and 15-generator system characteristics are given in [5], while those of the 110-generator system are

TABLE II
COMPARISON FOR THE 6-GENERATOR SYSTEM [5] WITH LOSSES

	Lambda iteration [1]	Proposed λ-consensus
p_1	447.5040	449.3094
p_2	173.3183	173.1754
p_3	263.4629	266.1296
p_4	139.0655	127.2407
p_5	165.4733	174.3958
p_6	87.1346	86.0221
Power (MW)	1275.96	1276.27
Losses (MW)	12.96	13.27
Net Power (MW)	1263.00	1263.00
Cost (\$)	15449.90	15452.09

TABLE III
COMPARISON FOR THE 15-GENERATOR SYSTEM [5] WITH LOSSES

	Lambda iteration [1]	Proposed λ-consensus
p_1	455.0000	455.0000
p_2	455.0000	455.0000
p_3	130.0000	130.0000
p_4	130.0000	130.0000
p_5	245.2997	298.2294
p_6	460.0000	460.0000
p_7	465.0000	465.0000
p_8	60.0000	60.0000
p_9	25.0000	25.0000
p_{10}	25.0000	25.0000
p_{11}	71.6401	44.9350
p_{12}	80.0000	56.4370
p_{13}	25.0000	25.0000
p_{14}	15.0000	15.0000
p_{15}	15.0000	15.0000
Power (MW)	2656.94	2659.60
Losses (MW)	27.62	29.60
Net Power (MW)	2629.32	2630.00
Cost (\$)	32546.25	32568.06

TABLE IV
COMPARISON FOR THE 110-GENERATOR SYSTEM [24] WITHOUT LOSSES

	Lambda iteration [1]	Proposed λ-consensus
Power (MW)	13000	13000
Cost (\$)	171446.61	171446.61

given in [24]. It should be noted that the conventional lambda iteration method is reported to be oscillatory in some cases [25], [26]. Authors' experience confirms this also for the 15-unit system. Therefore, the best solution among the oscillatory results is reported in Table III.

The solutions of the lambda-consensus algorithm and the lambda iteration method have almost the same cost in the 6- and 15-generator systems. In fact, the percentage difference of the corresponding costs is only 0.014% and 0.067%. Moreover, the solutions provided by both algorithms are the same when the transmission losses are neglected, as reported for the 110-generator system (the complete power distribution is not listed, but they are identical for both the algorithms). Thus, the performance of the proposed approach can be considered very satisfactory. In addition, the lambda-consensus has some advantages compared to the lambda iteration method. The proposed approach overcomes the need for a central authority that requires a high level of connectivity. In fact, the lambda iteration method requires the presence of an independent system operator (ISO) that broadcasts the current value of λ to all the generators, collects the corresponding powers from all the generators to update λ, and iterates the procedure. Moreover, the proposed algorithm requires local information and relies only on local interactions with the neighbors, leading to a sparse communication topology. This results in a more reliable structure compared to the star topology of the lambda iteration approach which is characterized by the presence of a single point of failure for the entire system. Furthermore, such a sparse topology can better and effectively accommodate plug-and-play features as shown in the previous sections. Finally, the proposed method can converge within a few iterations, given a proper gain chosen according to the design rules in Section IV-D.

VI. CONCLUSION

A distributed solution for the economic dispatch problem, incorporating transmission losses and power generator constraints, is presented. This approach reaches consensus on the Lagrange multiplier using a correction term that ensures the generation-demand equality. The proposed method relies on the distributed handling of the losses as well as the distributed estimation of the global power mismatch. The effectiveness of the proposed method is demonstrated by numerical simulations on several test systems with power generator constraints, transmission losses, and with small and large dimensions. Moreover, the proposed approach has shown very satisfactory performance when compared to the lambda iteration method, while offering advantages inherent to a distributed solution. Future works will extend this method to systems including renewable energy sources (e.g., wind energy) and energy storage units.

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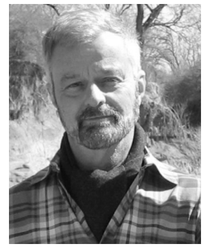

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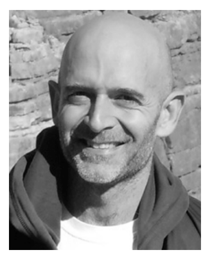

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