Fast λ-Iteration Method for Economic Dispatch With Prohibited Operating Zones

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***Abstract*—This letter presents a new method, the fast λ-iteration (FλI) method, to solve the economic dispatch (ED) problem considering the prohibited operating zones (POZs) and ramp rate limits of generation units. Necessary conditions for the optimal solution of the ED problem are presented and proved. The efficiency of the method has been verified on a 15-unit system and a Korea 140-unit system.**

***Index Terms*—Economic dispatch, fast λ-iteration, necessary condition, optimal solution, prohibited operating zone.**

I. INTRODUCTION

The prohibited operating zone (POZ) exists in a thermal or hydro generating unit due to the physical limits of power plant components, e.g., vibrations in a shaft bearing are amplified in a certain operating region. The POZs make the solution space of the economic dispatch (ED) problem discontinuous, and consequently most of mathematical programming methods fail to tackle this problem. This letter presents a fast λ-iteration (FλI) method to solve the problem.

Considering the constraints of the generation limits, the power load balance, the ramp rate limits, and the POZs, the ED model can be formulated as

$$\text{Min}: \quad F_I(\boldsymbol{X})=\sum_{g\in I} f_g(P_g)=\sum_{g\in I}\left(a_g+b_gP_g+c_gP_g^2\right) \tag{1}$$

$$\text{s.t.} \quad P_g^{\min}\le P_g\le P_g^{\max}, \sum_{g\in I}P_g-P_{\rm D}-P_{\rm L}=0 \tag{2}$$

$$P_g-P_g^0\le UR_g,\quad P_g^0-P_g\le DR_g \tag{3}$$

$$\begin{aligned} & P_g^{\min}\le P_g\le P_g^{1L} \text{ or } P_g^{1R}\le P_g\le P_g^{2L}\\ \text{or} \quad & \cdots \qquad \text{or } P_g^{Q_gR}\le P_g\le P_g^{\max}\end{aligned} \tag{4}$$

where $F_I(\boldsymbol{X})$ denotes the cost objective; N the number of generation units; $I = \{1,2,\cdots,N\}$; $\boldsymbol{X}$ a solution $(P_1,P_2,\cdots,P_N)$; subscript g Unit g; P_g the power output; a_g, b_g, and c_g the coefficients of fuel cost function; $P_g^{\min}$ and $P_g^{\max}$ the lower and upper power output limits, respectively; $P_{\rm D}$ the load demand; P_g^0 the initial power output; UR_g and DR_g the ramp up and down rate limits, respectively; Q_g the number of POZs in Unit g; P_g^{qL} and P_g^{qR} $(q=1,2,\cdots,Q_g)$ the lower and upper boundaries of the qst POZ of Unit g, respectively. $P_{\rm L}$ denotes the transmission line loss, which can be calculated based on the Kron's loss formula as follows [1]:

$$P_{\rm L}=\sum_{i=1}^{N}\sum_{j=1}^{N}P_iB_{ij}P_j+\sum_{i=1}^{N}B_{i0}P_i+B_{00} \tag{5}$$

where B_{ij}, B_{i0}, and B_{00} are B-coefficients.

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II. NECESSARY CONDITIONS AND THE FλI METHOD

In the following, two necessary conditions for a solution to be an optimal solution of (1)–(5) where $P_{\rm L}=0$ are given and proved:

1) If $\boldsymbol{X}^0 = (P_1^0,P_2^0,\cdots,P_N^0)$ is an optimal solution of the original model (1)–(5), then any sub-solution $\boldsymbol{X}^1 = (P_{i_1}^0,P_{i_2}^0,\cdots,P_{i_k}^0)$ where $\{i_1,i_2,\cdots,i_k\} = I_1 \subsetneqq I$ should be an optimal solution of (1)–(5) where I is replaced by I_1.
2) In an optimal solution of the original model, each unit's output should have the equal incremental cost value or be located at a generation boundary or be located at a POZ boundary.

Proof 1): Proof by contradiction, suppose that $\boldsymbol{X}^1$ is not optimal. Then there exists a sub-solution $\boldsymbol{X}^2$ such that $F_{I_1}(\boldsymbol{X}^2)<F_{I_1}(\boldsymbol{X}^1) \Longrightarrow F_{I_1}(\boldsymbol{X}^2)+F_{I-I_1}(\boldsymbol{X}^0)<F_{I_1}(\boldsymbol{X}^1)+F_{I-I_1}(\boldsymbol{X}^0) = F_I(\boldsymbol{X}^0)$. Thus there exists another solution having a smaller objective value than $\boldsymbol{X}^0$. There is a contradiction. □

Proof 2): The proof consists of two parts. **Part 1**: According to 1) and the equal incremental cost criterion [1], each unit's output in an optimal solution of the ED problem having no POZ should have the equal incremental cost value or be located at a generation boundary.

Part 2: The original model (1)–(5) can be decomposed into $\sum_{g=1}^{N}(Q_g+1)$ sub-models by restricting the power output of each unit to only one continuous operating zone. A sub-model can be expressed as (1)–(3), (5) plus only one of the (Q_g+1) constraints in (4). Then in each sub-model, each unit has only one continuous operating zone and has no POZ. An example of both the original model and the sub-model is plotted in subfigures (a) and (b) of Fig. 1, respectively.

According to Part 1 of Proof 2), for an optimal solution of the sub-model, each unit's output should have the equal incremental cost value or be located at a generation boundary. Considering that the generation boundaries of the sub-models are the generation boundaries or the POZ boundaries of the original model and that the optimal solution of the original model must be the optimal solution of one of the sub-models, we can come to the conclusion stated in 2). □

According to the necessary conditions given above, if a unit's output is located at a POZ, then it should be modified to a boundary of the POZ. The question is that which is its best output, the upper or lower boundary? For the convenience of illustration, the first derivative cost functions of 3 units are plotted in Fig. 1, where the coordinates of the 4 points marked are:

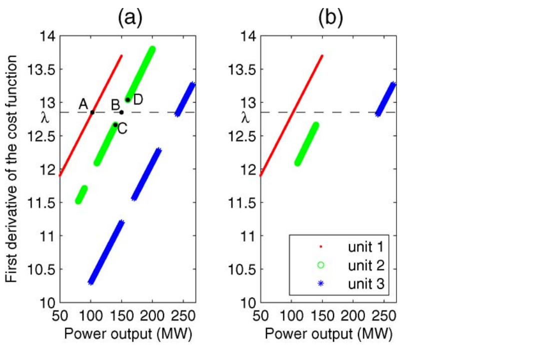

Fig. 1. First derivative of the cost function with respective to the power output for each of the 3 units. (a) Original model. (b) Sub-model.

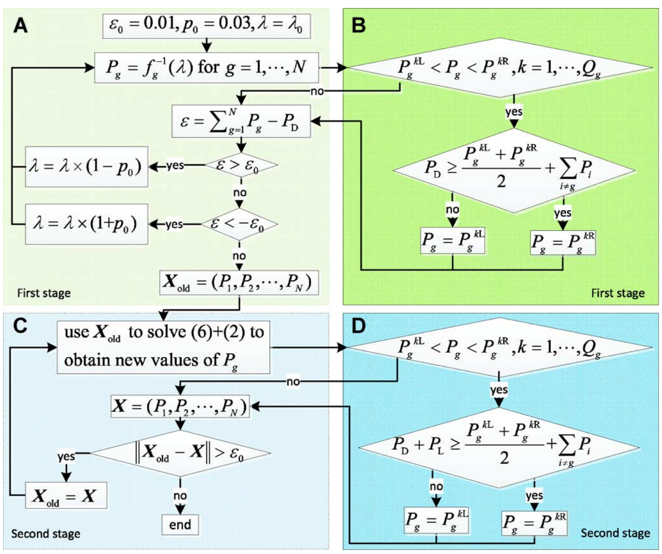

Fig. 2. Flowchart of the FλI method.

$\mathrm{A}(P_{10}, b_1{+}2c_1P_{10}), \mathrm{B}(P_{20}, b_2{+}2c_2P_{20}), \mathrm{C}(P_{21}, b_2{+}2c_2P_{21})$, and $\mathrm{D}(P_{22}, b_2 + 2c_2P_{22})$. Points A and B have the same first derivative cost values, i.e., $b_1 + 2c_1P_{10} = b_2 + 2c_2P_{20}$, and B is the middle of C and D, i.e., $2P_{20} = P_{21} + P_{22}$. In order to decide which of the upper and the lower boundaries of the POZ is better, two solutions of a 2-unit ED model are compared, i.e., comparing the total cost values of the 2 solutions $(P_\mathrm{D}-P_{21}, P_{21})$ and $(P_\mathrm{D}-P_{22}, P_{22})$: $f_1(P_\mathrm{D}-P_{21})+f_2(P_{21})-[f_1(P_\mathrm{D}-P_{22})+f_2(P_{22})] = a_1+b_1(P_\mathrm{D}-P_{21})+c_1(P_\mathrm{D}-P_{21})^2+a_2+b_2P_{21}+c_2P_{21}^2-[a_1+b_1(P_\mathrm{D}-P_{22})+c_1(P_\mathrm{D}-P_{22})^2+a_2+b_2P_{22}+c_2P_{22}^2] = (P_{21}-P_{22})[-b_1-2c_1P_\mathrm{D}+c_1(P_{21}+P_{22})+b_2+c_2(P_{21}+P_{22})] = (P_{21}-P_{22})[-b_1-c_1(2P_\mathrm{D}-P_{21}-P_{22})+b_1+2c_1P_{10}] = (P_{21}-P_{22})2c_1(P_{20}+P_{10}-P_\mathrm{D})$. Thus, if $P_\mathrm{D} > P_{20}+P_{10}$ ($P_\mathrm{D} < P_{20}+P_{10}$), then it is better for Unit 2 to be located at the upper boundary D (the lower boundary C); if $P_\mathrm{D} = P_{20}+P_{10}$, then either boundary can consist of the optimal solution.

Now we can give the details of the FλI that consists of two stages. In the first stage, a λ iteration is used to find the optimal solution of (1)–(5) where $P_\mathrm{L} = 0$. The flowchart of the first stage is given in Blocks A and B in Fig. 2. Block A is similar to the λ-iteration method which can refer to [1, Ch. 3.3]. In the second stage, the model (1)–(5) considering P_L is solved based on the solution obtained in the first stage. The necessary condition of an optimal solution can be expressed as [1]

$$\frac{\mathrm{d}f_g}{\mathrm{d}P_g} - \lambda\left(1 - \frac{\partial P_\mathrm{L}}{\partial P_g}\right) = b_g + 2c_gP_g - \lambda\left(1 - \frac{\partial P_\mathrm{L}}{\partial P_g}\right) = 0. \quad (6)$$

The collection, (6) plus the load balance constraint in (2), is known as the coordination equations. The coordination equations can be solved using an iterative procedure as given in Blocks C and D in Fig. 2. Block C is similar to the iterative procedure detailed in [1, Ch. 3.2]. Blocks B and D in Fig. 2 are used to deal with the POZs.

TABLE I
RESULTS OBTAINED BY DIFFERENT METHODS ON THE 15-UNIT SYSTEM

Method	$N_{\mathrm{Func\,Eval}}$	Time (s)	Cost (\$/h)
FλI	**26**	**0.015**	**32701**
FMINCON [4]	385	0.09	32882
PSO [2]	2×10^4	—	33020
AIS [5]	2×10^3	—	32854

TABLE II
RESULTS OBTAINED BY 2 METHODS ON THE KOREA 140-UNIT SYSTEM

$N_{\mathrm{unit,POZ}}$	Method	$N_{\mathrm{Func\,Eval}}$	Time (s)	Cost (\$/h)
4	FλI	**29**	**0.0221**	**1.65567**$\times10^6$
	FMINCON [4]	3525	1.391	1.6560×10^6
15	FλI	**26**	**0.0209**	**1.65569**$\times10^6$
	FMINCON [4]	3313	2.852	1.6763×10^6
30	FλI	**26**	**0.0237**	**1.65773**$\times10^6$
	FMINCON [4]	3348	4.878	1.7076×10^6

III. SIMULATION RESULTS AND ANALYSIS

The results of the ED problem on a 15-unit system [2] and a Korea 140-unit system [3] obtained by the FλI and other methods are given in Tables I and II, respectively. The initial solution of the FMINCON [4] is set to be $(P_g^{\min} + 1/N \times (P_\mathrm{D} - \sum_{g\in I} P_g^{\min}) + r)$ where r is a random number and its results in the tables are the average values of 1000 independent runs. The number of function evaluated (denoted as $N_{\mathrm{Func\ Eval}}$) and the time consumption are adopted to evaluate the computational load. It can be seen that the FλI can obtain the best results in terms of both precision and computational load and that the computational load of the FλI method is very low and it increases slightly as the unit number increases from 15 to 140.

The complexity of the ED problem will increase as the number of units having POZs (denoted as $N_{\mathrm{unit,POZ}}$) increases. To verify the efficiency of the FλI, we increase the $N_{\mathrm{unit,POZ}}$ from 4 to 15 and 30 with the results tabulated in Table II. We can see that the precision and the computational load of the FλI are not sensitive to the $N_{\mathrm{unit,POZ}}$, and its superiority to the FMINCON becomes more obvious as the $N_{\mathrm{unit,POZ}}$ increases.

IV. CONCLUSION

Based on the necessary conditions for the optimal solution, the FλI method has been developed to solve the ED problem considering POZs. Numerical simulation results have shown that the method performs very well in terms of both the precision and the computational load on both the 15-unit system and the Korea 140-unit system.

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